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George Paterson of
Castle Huntly Esq.





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Dr. GREGORY's
ELEMENTS
OF
CATOPTRICS
AND
DIOPTRICS.

Translated from the *Latin Original*,
With a large SUPPLEMENT,
By WILLIAM BROWNE, M.D.

THE SECOND EDITION.

To which is added,
An APPENDIX,
By J. T. DESAGULIERS, LL. D. F. R. S.

CONTAINING,

An Account of the REFLECTING TELESCOPES;
and of the Inventors, Improvers, and Imitators of them,
till they were brought to Perfection by JOHN HADLEY,
Esq; Vice-President of the *Royal Society*.

With Original LETTERS which passed between Sir
ISAAC NEWTON and Dr. JAMES GREGORY, relating
thereunto.

NOW FIRST PUBLISHED.

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P R E F A C E

(To this SECOND EDITION)

By Dr. *DESAGULIERS*.



IT is almost twenty Years, since Dr. BROWNE desired me to look over his Translation of this Treatise of *Catoptrics* and *Dioptrics*, and his own large *Supplement*: I found the Translation to agree with the Original, and the *Addenda* very proper for such Persons as apply themselves to *Optics*; and accordingly allowed him to mention my Approbation of the Work.

Now the Book being out of Print, and a great Demand made for it, the Doctor was requested, by Mr. CURLL, to publish a new Edition of it: But as he lives at *Lynn* in *Norfolk*, he could not, as he wrote him Word, fix any certain Time for the Publication; he therefore has desired me to do it for him, al-

lowing me to make such Emendations and Additions as I should think fit. I find no Occasion for *Emendations*, except a few Errors of the Press, and in the Plates, which I have corrected: But I have made some *Additions* in his *Introduction* and his *Supplement*; and have, by way of *Appendix*, given a full Description of the two Sorts of *Reflecting TELESCOPES*, at present so much in Vogue, and so justly valued; which I have explained by several Figures in a *fourth* Plate.

The Reader will find who have been the Inventors and Imitators of these *Telescopes*, by Sir ISAAC NEWTON'S and Dr. JAMES GREGORY'S * *LETTERS*, written near sixty Years ago; so much of which I have transcribed from the Originals (now in the Possession of WILLIAM JONES, Esq;) as relates to these Matters: And the Public are wholly indebted for the Use of them to that excellent Mathematician and ingenious Mechanic JOHN HADLEY, Esq; Vice-President of the *Royal Society*.

As it may be expected that I should give some Account of our Author, I have here subjoined what ANTHONY à WOOD says of him (in his *Fasti Oxon.* Vol. II. Page 225.) viz.

* Uncle to our Author, who wrote the *Optica promota*, and published it in 1663.

“ DAVID GREGORY, M. A. of the Uni-
 “ versity of *Edinburgh*, was admitted to the
 “ Rectory of *Brightwell*, near *Wallingford*
 “ in *Berkshire*, 1691. The same Year he
 “ accumulated the Degrees in Physic, and
 “ became a Master-Commoner of *Baliol-*
 “ *College*.

“ This Gentleman, who was born at
 “ *Aberdeen*, and mostly educated there,
 “ hath extant, *Exercitatio Geometrica de*
 “ *Dimensione Figurarum; sive, Specimen*
 “ *Methodi generalis dimetiendi quas-*
 “ *dam Figuras. Edenb. 1684. qu.* At
 “ which time he was Mathematic Professor
 “ there.

“ His Uncle, Mr. JAMES GREGORY,
 “ printed at *Padua*, in the Year 1667; a
 “ Book entituled, *Vera Circuli & Hyper-*
 “ *bolæ Quadratura*.

“ In the *Philosophical Transactions*,
 “ N^o. 207, Jan. 1693, is, *Solutio proble-*
 “ *matica Florentini de veliformi quadra-*
 “ *bili*, by our Author DAVID GREGORY,
 “ then Fellow of the *Royal Society*.

“ In the *Philosophical Transactions*, N^o.
 “ 214, is, *An Epistle from Dr. GREGORY*
 “ to

“ to Sir ROBERT SOUTHWELL, as-
 “ certaining some *Mathematical Inven-*
 “ *tions* to their AUTHORS. Dated Nov. 15,
 “ 1694.

“ He published also, CATOPTRICÆ
 “ & DIOPTRICÆ *Sphericæ Elementa.*
 “ *Oxon.* 1695. 8vo.” A Translation of
 which being what is here undertaken. And
 thereunto are added,

I. A Method for finding the *Foci* of all
Specula, as well as *Lens*’s, universally.
 As also for *Magnifying* or *Lessening* a
 given *Object* by a given *Speculum* or
Lens, in any assigned Proportion, &c.

II. A *Solution* of those *Problems* which
 Dr. GREGORY has left undemon-
 strated.

III. A particular Account of MICROSCOPES
 and TELESCOPES, from Mr. HUYGENS.
 With an *Introduction*, shewing the Dis-
 coveries made by CATOPTRICS and
 DIOPTRICS. By Dr. BROWNE.

*A LETTER from Dr. BROWNE
to Mr. CURLL.*

S I R,

Lynn, July 6, 1734.

I Received yours: GREGORY was the Child of my Youth; you was so good as to take it off my Hands, and undertook to bring it up and maintain it at your own Charge: So that it is now more properly yours than mine. As to a new Edition of it, I cannot engage to do it by any particular Time, because our leisure Hours are so very uncertain: So that if you could get the same thing done by the Gentlemen you mention (Dr. DESAGULIERS, or Mr. JONES) which must be worth your while, they are so much more capable of doing it than my self, that, with the Recommendation of their Names, I apprehend the Work would meet with a much more favourable Reception from the Public; and secure, in consequence, a larger Share of what you may expect from it your self. Very sure I am, that your publishing only a correct Edition of the self-same Work, without Alterations and Augmentations, will never answer in any respect. But judge for your self; and whatever it is you undertake, I heartily wish Success; only, if you will take my Advice, don't grudge the compleating it on the best Terms you can, by either of the Gentlemen you named. I am

*Your old Friend, and
Humble Servant,*

W. BROWNE.

*A Second LETTER from Dr. BROWNE
to Mr. CURLL.*

Dear SIR,

Lynn, Nov. 2, 1734.

I AM glad you have put Dr. GREGORY into so much better Hands than mine. I have nothing to add; nor would I by *any means*, by *any thing* of mine, diminish the Value of an Edition, which will be much more esteemed, in *being republished by Dr. DESAGULIERS.*

I am,

(With Service to Him)

His and your humble Servant,

W. BROWNE.

Dr. BROWNE'S TWO LETTERS to Mr. CURLL, shew that every thing has been done in Concert with That Gentleman.

J. T. DESAGULIERS.

*Channel-Row,
Nov. 21, 1734.*



INTRODUCTION;

Shewing the *Discoveries* made by

CATOPTRICS and DIOPTRICS.

By Dr. BROWNE.



It is a great Encouragement for those who would take the Pains to perfect themselves in any Science, to be first informed of what Service it will be to understand it: And since there is no Part of Learning of so real and general Benefit to Mankind, as this of *Catoptrics* and *Dioptrics*, it is but Justice both to the Subject and Reader, to give some Account of the many wonderful Discoveries which we owe entirely to this Science.

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The

The *Sight* of Man is of it self confined to very narrow Views; and though it takes in a great part of the Creation at once, yet all is represented in Miniature and imperfectly. The naked *Eye* sees only so much of external *Objects* as is sufficient to move the principal Passions, and give notice of what more immediately concerns the Safety and Happiness of the Animal. What is more than this, was left as a Subject for our Curiosity, upon which we might exercise those Faculties which are bestowed by our bountiful Creator for this very End, of searching into the astonishing Mechanism of all his Works, and from thence enlarging our Idea of his Greatness. *Objects* placed at a great Distance, whether upon the Surface of our Earth, or in the Heavens, are seen under so small an Angle, that their Parts are not to be distinguished one from another; and by this means those distant charming Scenes of Nature were hid from us, which the Study of *Catoptrics* and *Dioptrics* has since laid open to our View. This noble Part of Knowledge teaches us how, by a due Position of *Glasses* ground into certain Figures, we may enlarge the Diameters of the heavenly Bodies, and all such *Objects* to which we are allowed no nearer Approach, in what Proportion we please,
and

and view them as perfectly and distinctly as if we could summon them before us, and command them to the End of our *Telescope*. This has brought us into a perfect Acquaintance with those surprizing Parts of the Creation, which are far separate from this Globe of ours, and with which we are allowed no Commerce but Looking. We can now perceive the *Sun* to be a vast Globe of Fire, and by the different *Phases* of all the *Planets*, that *he* is the Fountain of all their *Light*. The Surfaces of most of them appear like so many *Maps* of Land and Water, and there are few now but allow both them and the *fixed Stars* some nobler Use than to twinkle upon us o'Nights. By fixing upon some remarkable *Spots* on their Surfaces, and observing how they shift their Position, and in what time they again return to the same Place, we determine the Motion of these Bodies round their *Axes*, and the Time in which that Revolution is performed. Several *secondary Planets*, or *Satellites*, which were too small for the naked *Eye*, are now discerned to move round *Jupiter* and *Saturn*, as the *Moon* round our Earth; and about the last of them is seen the particular *Phænomenon* of an *Annulus*, or *Ring*. Nor is the Discovery of these *Satellites* merely speculative, but of prodigious

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gious Use and Advantage; for their *Eclipses* have determined the *Velocity* of *Light*, as *Romer* has shewn,* and are so frequent, as to be the most constant Appearance the Heavens afford us at present, for the Solution of the great and valuable Problem of the *Longitude*. The Distances, Magnitudes, and Motions of all the heavenly Bodies, and even the Irregularities of the *Moon*, have by this means been so nicely observed, and by the Power of Numbers reduced within some few Tables for common and easy Use, that their Places for any determinate Instant of Time to come are now to be predicted as easily, and almost as exactly, as we could wish.

The nice Prediction Dr. HALLEY gave of the late *Solar Eclipse*, total at *London*, (a rare Sight in our Part of the *Globe*) is such an Instance of the great Perfection to which we are arrived in these Matters, as has amazed those unthinking Gentlemen, who were only to be rowzed out of their Security in Ignorance by the Apprehensions of *Doomsday*. The prodigious Distances

* Since that Time the Velocity of Light has been more exactly determined by the Observations of the Reverend and Learned Mr. BRADLEY, Savilian Professor of Astronomy at Oxford; who shews, that Light comes from the Sun to us in 8 Minutes and 13 Seconds.

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of the *fixed Stars* beyond that of any of our *Planets*, is, besides their little or no *annual Parallax*, plainly deducible from the *Telescope*,* for the longest that ever was made, and which perhaps represents their *apparent Diameters* one or two hundred times larger than the Truth, has been so far from magnifying them, that, by cutting off those irregular Rays which hinder us from distinguishing the true Termination of their Orbs; it makes them appear something lessened; besides that Mr. HUYGENS has given a Method of even computing these Distances by means of the *Telescope*.

It is now reckoned no absurd Notion to conceive these *fixed Stars* as so many *Suns*, probably at as great Distances one from another as they are from us, and every one to have their *System* of inhabited *Planets* circling round it: And perhaps the Number of those which we see, counted by HEVELIUS to be 1888, may bear little or no Proportion

* The curious Observations of the Learned Professor BRADLEY abovementioned shew, that the fixed Stars are much farther from us than was imagined even by those who supposed that Light must be six Months in coming from them to the Earth: Whereas by Mr. BRADLEY's Observations they must be 40,000 times farther from us than the Sun; and consequently Light will be above six Years coming from the fixed Stars to the Earth. Whence it follows, that if it should please GOD to annihilate one of the fixed Stars, we should continue to see it six Years after it was out. See Philos. Transact. No. 406.

to those others that may be dispersed thro' the vast Regions of the *Universe*, at such Distances from our little *Ball*, that no Assistance can ever help us to a Sight of them. A Notion that gives surely the most just and noble Sentiments that the Mind of Man can entertain of an Almighty Author! That the *milky Way* in the Heavens, which we behold in a clear Star-light Night, is nothing else but a continued Cluster of such *fixed Stars*; is a Truth of which we are assured by the *Telescope*. And to the same Help it is we owe all we know of those heavenly Bodies called *Comets*; their Distance, Magnitude, and Motion round the *Sun* in such *eccentrical Orbits*, as come some of them very near to right Lines. To what a surprizing Height this *cometical Astronomy* has been carried by the present Age, notwithstanding the Observations we have been able to make upon these Bodies are so few, and those made by our Predecessors so imperfect, may be seen in the Writings of those incomparable Astronomers, Sir ISAAC NEWTON, Dr. GREGORY, and Dr. HALLEY. That in every clear Morning and Evening we see the *Sun* for some time before he rises, and after he sets, is a *Paradox* only to be unriddled by *Dioptrics*: And if we would know the true Place of any heavenly Body

elevated

elevated not many Degrees above the *Horizon*, the same Science tells us, that here *seeing* is not *believing*, but that we must correct our *Eye-sight* by a *Table of Refractions*. It is true, the *Ratio* of Refraction of the *Atmosphere* very near the *Horizon* does not observe a constant Rule, because there happens a very great Variety in the Accumulation of Vapours about those Parts: But then this Variation depends pretty regularly upon the Position of the *Sun* above or below the *Horizon*, and the different State of the Weather; and if in the Morning or Evening we see the lower Parts of a distant Tower or Mountain through a *Telescope* fixed in Position, we shall find the upper Parts of the same Tower or Mountain in the same Place, if our Observations be made nearer Noon, and just at Noon the same Object will be seen lowest of all, as the accurate Mr. HUYGENS has observed: And this Difference is greater in cold and moist, than in hot and dry Weather; and though not in a Proportion always certain, yet constant enough for physical Purposes. The *Crepusculum* or *Twilight* is determined from the Rays of the Sun below the *Horizon*, first refracted at their Entrance into the Earth's Atmosphere, and then reflected from that Part of it near our *Horizon*, or rather
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from the contiguous Surface of the *Æther*, as from a *concave Speculum*: And the Height of the *Atmosphere* has been attempted from this *Theorem* by VARENIUS, but the Air being a *Medium* of different Density, and consequently of different Refraction at different Distances from the Earth, refracts the Rays of the *Sun* into *Curves*, and makes that Solution less exact. In short, without the Assistance of *Telescopes*, *Astronomy* could have come to nothing, and our Observations of the Heavens had gone little further than foretelling a fine Morning from the Setting of the *Sun*, or a Shower of Rain from the Course of the Clouds. These Instances are sufficient to shew that all the noble Discoveries of the Heavens, of which the present Age may so justly boast of, are derived from the Knowledge of *Catoptrics* and *Dioptrics*; and whatever Improvements are hereafter to be made, can be expected from no other Fountain.

I shall now descend to a Prospect no less amazing, which the same Science opens to us in the minute Parts of the Creation. The Difficulty which hindered the naked *Eye* from examining the smallest Particles and subtle Texture of those Bodies that are always under our Command, was, that when such *Objects* are brought near enough the
Eye

Eye to have their least Parts subtend a sensible Angle, they become without the Limits of *distinct Vision*. For as long as the *Pupil* of the *Eye* can, by the circular Fibres of the *Uvea*, be contracted in proportion as the *Object* is brought nearer, the *Cones of Rays* from each *Point* may still be looked upon as *Cylinders*, and will consequently be brought to a Point in the *Focus* of the *Eye*, which is at the *Retina*, and still make *distinct Vision*: But this Contraction of the *Aperture* of the *Pupil* holding no nearer than about four Inches from the *Eye*, if the *Object* is brought nearer than this, the encreased Magnitude is of no further Service, because the *Rays* from each *Point* must be now considered as *diverging*, and will consequently after Refraction at the *Eye* be made to converge to a *Focus* beyond the *Retina*, (which is the Place of the *Focus of parallel Rays*) and so make *confused Vision*, and the nearer the *Object* approaches, the farther is its *Image* projected beyond the *Focus* of the *Eye*, and becomes so much the more confused. *Dioptrics* teach us to remedy this Inconveniency two ways, the first is, by looking through an *Hole* pricked in a thin *Plate*, suppose of *Lead*, whose *Aperture* must be so much the smaller as the *Object* is nearer, for this supplies the Place of a farther

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ther Contraction of the *Pupil*: But because this lessening the *Aperture* excludes a great many *Rays* from each *Point*, and so diminishes the *Brightness* of the *Image*, and that in a *duplicate Ratio* of the *Diameter* of the lessened *Aperture*, the same Science has also pointed out to us the more curious Invention of the *Microscope*. By means of this we discern the admirable Range of the *constituent Particles* of all such Bodies as come within our nearer View and Acquaintance. The *Cuticula*, or outward Skin of the human Body, is found to be composed of several *Strata* of *Scales*, lying one over another in different Numbers, according to its different Thickness in different Places: Between these *Scales* the *miliary Glands* dispersed over the Surface of the whole Body are seen to send out their *excretory Ducts*, through which we perspire; and about one of these *Scales* the *Microscope* reckons near 500 such *Ducts*, and that one Grain of Sand will cover 250 such *Scales*: So that one Grain of Sand will cover 125,000 *Orifices* of these *excretory Ducts*. A Discovery that must make us bless our selves, and stand astonished at the *Infinity* of the *Creator*, when the Creature is so much beyond our Comprehension! The inquisitive Mr. LEWENHOEK has obliged the World with
a pro-

a prodigious Number of such surprizing Truths, which the curious Reader will find among his Writings. The extreme Ductility and Minuteness of the Particles of *Gold* is no less wonderful; for a Piece of Silver gilt with *Leaf-Gold*, and drawn into the finest gilt Wire, whose Diameter is $\frac{1}{386}$ of an Inch, and the Thickness of the Skin of *Gold* (as Dr. HALLEY has, from the specific Gravities of the two Metals, computed it) not above $\frac{1}{134500}$ of an Inch, discovers not the least Particle of Silver through the Pores of this Skin of Gold, though viewed by the *Microscope*. The Particles of the Dust which flies like Smoke out of the *Fungus pulverulentus*, or *Puff-Ball*, when burst, are discerned to be perfect Spherules of an Orange-colour, something transparent, and their Diameters not above $\frac{1}{50}$ of that of an Hair: So that a Tube of an Hair's Breadth would contain 125,000 such Spherules. The *Circulation of the Blood*, that noble Discovery of our Immortal HARVEY, is now made visible in the transparent Parts of Animals, such as the Fins and Tails of Fishes, and the Feet of Frogs; and the *Anastomoses* of the Arteries and Veins put out of Question. It is no less instructive and curious to behold the different Organization of the lesser *Species* of Animals, as the regular

Armour of the *Flea*, the jagged *Proboscis* of the *Tick*, and the Bristles of the *Mite*; and in a *Louse* as he stirs his Legs, you see the Motion of the *Muscles* of his Body, whose *Tendons* seem all to be united in a longish dark Spot in the middle of his Breast, and the like Motion is observable in the *Muscles* of the several Articulations of the Legs, and in those of the Head, as he stirs his Horns, there also appears a great Variety of Branchings of Blood-Vessels, and the Pulse regularly beating in several Arteries, and even the *peristaltic Motion* of the *Intestines*, continued from the Stomach all the way to the *Anus*, which is also to be seen in the *Flea*, and several sorts of transparent *Maggots* and *Caterpillars*. Besides these, the *Microscope* has presented us with an infinite Variety of little *Animals*, with which the naked Eye can have no Acquaintance. They are observable in different Shapes and Sizes about the green Weeds growing in Water, in several aromatic Infusions, and in the standing Water in the Hollow of the *Cabbage* and *Teazle*, but in such Numbers in that which drains from an Horse-Dunghil, that they appear sometimes as thick as Bees in a Swarm, or Ants on an Hillock, and must be diluted with fair Water to separate their different *Species*.

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The *Animalcula* in the *Semen virile* are, of all the Subjects, most worthy our Notice and Admiration, because from this little shapeless Creature we have Reason to believe that the glorious Frame of Man himself arises, and this the rather, because in the Seeds of Plants and Trees the *Microscope* discovers the future Plant and Tree already formed, and the *Semen masculinum* of other Animals, as Bucks, &c. are found to be furnished with its *Animalcula*: Where it is to be remarked, that sometimes the Viscidity of the *Semen* hinders the Success of Observations of this kind, and must in such Cases be diluted with a little warm Water. This Theory of Generation is handsomely and at large explained in the *Philosophical Transactions* by Dr. GARDEN. The Use of *Microscopes* has found that loathsome, catching Distemper the *Itch* to be occasioned by the Depredations made upon the Skin by a certain *Species* of voracious *Animalcula*, which are described in the *Philosophical Transactions* by a Foreigner in a Letter to Dr. MEAD; and indeed seems to promise the finishing Hand to the Science of *Medicine*: For if we can once, by a sufficient Number of Experiments, determine the different Change of the Texture of the Blood in every different Distemper from that which it enjoys in its
natural

natural and healthful State, and by mixing the smallest Particles of several Medicines with it, find out those which will again reduce it to that natural State, there seems to be nothing more wanting to the *practic Part*; and if the true *mechanic Theory* of all these different Changes be ever to be known whilst we live in this Cloud of Flesh, I'm sure we must have the *Data* for it from the *Microscope*. The Method of estimating the Magnitude of *microscopical Objects* seen by a single *Lens* only, being so easy that any one who knows ever so little of *plane Trigonometry* will easily hit of it himself, is not mentioned in this Book; besides that it is already given by Dr. KEILL in his *Physical Lectures*; where he shews that an *Animalculum* placed at the Distance of $\frac{1}{10}$ of an Inch before a single *Lens*, and seen through it under an Angle of one Minute in Length, is nearly $\frac{1}{1000000}$ of an Inch long; and if its Figure were cubical, the Magnitude of it would be $\frac{1}{1000000000000000000000000}$ of a cubick Inch. From whence he concludes with a great deal of Reason, that what some philosophical People dream of Angels, may very well be applied to these *Animalcula*, that when they have a mind to be merry, several thousand Couples of them may lead up a Country-Dance upon the Point of a Needle.

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I am sensible I need say no more in Recommendation of the Subject: And for so much of the Book as is Dr. GREGORY's, the very Name of the Man gives it sufficient Reputation. But I am conscious that Part which I have attempted to add stands in need of some Name to recommend it with which the World is much better acquainted than with mine: And for that Reason I have obtained the Favour of making use of those of Dr. DESAGULIERS and Mr. JONES; Gentlemen against whose Judgment in these Matters their Approbation of the following Papers is the only possible Objection; and whose Names can never fail of meeting with that Esteem which they deserve, when fixed to any thing of their own, however they may happen to be treated for appearing in this Place to recommend what is mine.

W. BROWNE.



P R E-



PREFACE.



THESE Elements of Catoptrics and Dioptrics, which were eleven Years ago read publickly in Lectures, in the University of Edinborough (1684) I have composed for the Use of young Students, in such a manner, that nothing but EUCLID's Geometry is required towards the understanding them. For though I have likewise demonstrated from higher Principles, why Spheres and Conoids observe the same Laws, both in reflecting and refracting Light; yet those who are solicitous only about the Properties of Plane and Spherical Surfaces, may, without the least Inconveniency, pass over all that. These last are what we have more especially considered, as also such Optical Instruments as are made by a
a Combina-

P R E F A C E.

Combination of them ; that is, whose Effects arise either from a single Lens or Speculum, or from several combined together. I have, as KEPLER did before me, made use of some Postulata, that come not quite up to Geometrical Strictness, but yet are of great Service in resolving Questions in Natural Philosophy, which would otherwise be extremely intricate. If these Elements be found capable of instructing such as are less conversant in Optics, I shall have my End.

Oxon. 1695



I N T R O -



INTRODUCTION.



THOSE Questions concerning the Nature of LUCID BODIES, and of LIGHT, which usually cost Philosophical Writers so much Pains and Trouble, we have, after the Example of Mathematicians, omitted. For if they, who by their Inventions, have so much improved this Science, had employed all their Time in enquiring into the absolute Nature of its Object, and the most hidden Causes of its *Phænomena*, not contented with deducing after a Geometrical Manner from those more simple and easily observed Properties of Light, others less obvious; Optics had fallen much short of that Perfection to which they are now arrived. Therefore whether Light be the Action of the lucid Body driving on those Bodies that lie next it, which likewise drive on others next to them, and so on of the rest, none of them in the mean time singly moving any considerable Space; or whether it consists, which is much more likely, of Corpuscles projected with a very great Velocity from the lucid Body thro' the circumambient Spaces; or whether it be of a quite different Nature; and such as

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may

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may hereafter, or perhaps never, be perfectly discovered ; yet we may easily be allowed to assume this Property of it, which is simple enough, and confirmed by Experiments, That from every lucid Point, Rays are every way propagated in an Orb, and, in a Medium that is homogeneous, are diffused in right Lines (such being the *shortest*) after the same uniform Tenour.

But if those Rays meet with a Medium differently affected, whose Parts either strike them back, or diffuse them more or less than the Parts of the former Medium did, they will then suffer an *Inflexion*, by which general Name, I would, with other Authors, understand their Reflexion, as well as Refraction. For Light striking upon a Surface, that absolutely denies it Entrance, but yet hinders not its being diffused after the same manner as before, will all of it return back the easiest way it can find, diffusing itself still as at first ; this is called *Reflexion* of Light, and the Science which treats of the Laws it observes, according to the different Incidence of Rays upon Bodies of different Figures, is called *Catoptrics*. But if the Medium, upon which the Light strikes, allows indeed a Passage to its Rays, but then so as that they must be either more or less diffused than before, every Ray will be inflected from

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from the right Line, in which it was before disposed to proceed, and this Inflexion is called *Refraction*; and the Science which demonstrates the Laws and Effects of it, is called *Dioptrics*.

The *Radiant* is that, from every Point of which Rays are propagated.

Parallel Rays, are such as are equidistant from one another.

Diverging Rays, are such, as if produced both ways, meet on the Side contrary to that towards which they move.

Converging Rays, are such as, if produced, meet on the same Side towards which they move.

It must be observed, that this *Parallelism*; *Divergency*, and *Convergency*; is to be understood of Rays proceeding all from the same Point.

The *Focus* is that Point, in which Rays proceeding from the same Point of the Radiant, being produced, do meet; whence the *Focus* of parallel Rays is looked upon as infinitely distant.

The

The *Angle of Incidence*, is that which is made by the incident Ray, and a right Line perpendicular to the inflecting Surface at the Point of Incidence.

The *Angle of Reflexion*, is that which is made by the reflected Ray, and the same Perpendicular.

The *Angle of Refraction*, is that which is made by the refracted Ray, and the same Perpendicular produced.

The two following Propositions we have assumed for *Axioms*, because they agree both with Geometrical Reasoning and Experiments.

A X I O M S.

I. A Ray of Light falling perpendicularly upon an inflecting Surface, either proceeds directly forward, or is reflected back upon itself. For since the Direction of the Ray to the inflecting Surface is, of all that can be drawn from the radiant Point, either the *least*, if the inflecting Surface be a Plane, or perhaps, where this Circumstance is wanting, the *greatest*, and in both Cases a determinate, and *only one* of its kind, the Ray will still persist in the

the same Direction, either proceeding forward, or returning backward. For there are innumerable right Lines inclined to this *only one* in any given Angle, no one of which can consequently claim to its self the Direction of the Ray with greater Justice than the rest.

2. If a Plane be supposed, produced thro' the incident Ray, and a right Line perpendicular to the inflecting Surface at the Point of Incidence, the inflected Ray will likewise be found in the same Plane; or, which is the same thing, every Inflexion is made in a Surface that is perpendicular to the inflecting Surface. And this Surface shall, according to ALHAZEN, be called the *Plane of Inflexion*. For since this Plane is either the *least* or *greatest* of all that can be produced thro' the radiant Point, to the inflecting Surface, and consequently an *only one*, the Proposition is demonstrated after the same manner as the former: And indeed if we more closely consider it, we shall find the former to be only a particular Case of this latter.



THE ELEMENTS OF CATOPTRICS.

PROP. I. THEOR. I.

IF a Ray of Light be reflected from a plane Surface, the Angle of Reflection will be equal to the Angle of Incidence. [Plate 1. Fig 1.]

Tho' the Demonstration of this Theorem belongs more properly to Physics, and a Mathematician might very well take it for granted, as sufficiently proved by Experiments; yet in Compliance to Custom, we shall here give a Demonstration of it, and That such a one, as shall have no Dependence upon any Party of Philosophers whatever.

Let AB signify a Ray proceeding from the Radiant Point A , and falling upon the plane reflecting Surface, GC at B . I say its reflected Ray BE will be such, that the Angle of Reflection PBE (BP being supposed perpendicular to GC) will be equal to the Angle of Incidence PBA .

From the Radiant Point A , let fall AC perpendicular to the Surface GC . All the Rays comprehended between AB and AC , and which possess, in the reflecting Plane, the Right Line BC (the common Section of the Plane of Inflection with the reflecting Plane) would, if the Medium GC *de* were penetrable to *Light* and homogeneous with the former $EBCD$, be diffused in the same constant Tenour; that is, at the Distance Be beyond GC , they would be diffused into *de*, ABe and ACd being supposed Right Lines. And since, by Supposition, the plane reflecting Surface BC , neither encreases nor diminishes the Diffusion of those Rays (for if it were otherwise, which no Experiments yet give us any reason to suspect; it is plain the Angle of Reflection as well as of Refraction would, for the same Reasons, become unequal to the Angle of Incidence: For the due inflecting of which Rays, CARTESIUS, has contrived such Conoids as are proper, (*Geom. lib. 2*) but only strikes them back upon the same Medium in which they were before;

fore; that is, upon a Medium where the Rays are disperfed or diffused after the same manner as at first; it is plain, that after they are reflected from the Surface GC , the Rays will be disperfed after the same manner, as they would have been, if they had never met with the Surface CG , but, (that being removed,) had still remained in the same Medium. In which case, when the Distance is increased by the Excess Be , the Rays would be disperfed thro' the Space ed . Wherefore it is plain, that at the Distance BE equal to Be , they will be dispers'd thro' the Right Line DE , equal to the Right Line de . But the Ray AC is reflected upon it self, wherefore DE is equal to de , and the Figure $BCDE$ every way similar and equal to the Figure $BCde$; and indeed is the very same reflected, or revolved half way about the fixed Right Line BC . Therefore if from the equal Angles $EB C$, $e B C$, the right Angles $P B C$, $A B C$ be taken away, the Angles $EB P$, $e B p$ will remain equal. But $AB P$ is equal to $e B p$, therefore the Angle of Incidence $P B A$, is equal to the Angle of Reflection $P B E$.
Q. E. D.

SCHOLIUM.

In laying down this Law of Reflection, whereby the Angles ABP , EBP , are made equal, great Respect is had to the *Maximum* and *Minimum*. For the Sum of the Right Lines AB , BE is a *Minimum*; that is, less than the Sum of any others drawn from the same Points A and E , to any other Point of the Plane GC : and so *vice versa*. For it is requisite that the Course which the Ray (reflected from the Plane GC) takes from A to E , should be the shortest of all. Because *Nature ever acts by the most easy and expeditious Methods*.

But if the Reflection be from a curve Surface, the aforementioned Sum of Rays (or perhaps their Difference) is sometimes a *Maximum*. For Mathematicians know how near the Relation is between a *Maximum* and a *Minimum*, between the Sum and the Difference, and how easy the Transition is from one to the other.

COROL. I.

Hence the Angles EBG , ABC (which are likewise by some call'd the Angles of Incidence and Reflection) are also equal: for they are the Complements of the former to right Angles.

COROL.

COROL. 2.

If the Reflected Ray be looked upon as an Incident Ray, the Incident Ray will *vice versa*, be its Reflected Ray.

COROL. 3.

If the Angles PBE , PBA , or the Angles GBE , CBA be equal, BE will be the Reflected Ray belonging to AD .

PROP. II. PROBL. I.

THE Focus of Rays falling upon a plane Speculum being given, to find the Focus of those Rays after their Reflection from the Surface of the Speculum. [Plate 1. Fig 2.]

From the given *Focus* A , draw the Right Line AC perpendicular to the plane Speculum CF , which produce to a , that Ca may be equal to CA ; a will be the *Focus* required. Let AD be an incident Ray, join aD , and produce it.

Because in the Triangles ACD , aCD the Sides AC , aC , are equal, and the Angles ACD , aCD also equal, because both right, and the Side CD is common to both, therefore the Angle ADC is equal to aDC ; but aDC is equal to EDF ; therefore ADC ,
EDF

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EDF are equal. And consequently DE, by *Corol* 3. *Prop.* I. is the Ray reflected from the plane *Speculum* CF, belonging to AD. The same may be shown of any other Ray proceeding from the Point A. and falling upon the *Speculum*. From whence it is plain, that the Rays diverging from A, after their Reflection from the plane *Speculum* CF, will diverge from the Point *a*: that is, those Rays, whose *Focus* before Reflection was A, will after their Reflection have *a* for their *Focus*. *Q. E. D.*

SCHOLIUM.

Since the Eye in any Position, as at O, will receive the Rays thus reflected, after the same manner, as if they really had proceeded from the Point *a*, it is plain the Image must appear in that Place: Because the Rays diverging from the Point *a*, affect the Eye after the same manner, as if that were the primary Radiant.

COROL. I.

From this *Prop.* and *Corol.* 2. *Prop.* I it follows, that the Rays ED, OB converging towards the *Focus* *a*, will after their Reflection from the *Speculum* CF, converge towards the *Focus* A: and that such a Position of the Plane CF, may be easily

easily assigned, as shall make the Angle OBA equal to any given Angle; which is done by taking OB equal to half the Complement of the given Angle to two right ones.

COROL. 2.

In a plane *Speculum*, the Image of any radiant Point is seen in that Place where the Reflected Ray OB which passes thro' the Centre of the Eye, meets with the perpendicular AC , let fall from the radiant Point upon the *Speculum*. Whence, tho' every part of the *Speculum*, except B , were covered or taken away, the Image would nevertheless be visible: and if the *Speculum* at B be covered, tho' all the rest be open, the Image will not be seen at all.

COROL. 3.

The Images of Objects that are inclined to the Plane of the *Speculum*, are inclined to the plane of the *Speculum* after the same manner as the Objects themselves; and therefore in a plane Horizontal *Speculum*, vertical Objects appear inverted.

This and what follows will be easily understood, if the Object be conceived as made up of several radiant Points; and the Image of every one be attended to: For of all these together the Image of the Object consists.

COROL.

C O R O L. 4.

A radiant Plane and its Image made by a plane *Speculum*, are equal and similar Figures, but not similarly placed. They differ as the Right Hand from the Left, or as a Figure engraven upon a Copper Plate, does from the Impression of it taken upon Paper. For they will fit if they come together.

C O R O L. 5.

Because the Right Line aB is equal to AB , aO will be equal to AB and BO together; that is, the Distance of the Image from the Eye, is equal to the Incident and reflected Ray taken both together.

C O R O L. 6.

Whatsoever has been said of the Image of any Object or primary Radiant, holds true also of the Image of another Image. From whence arises that Multiplication of Images, made by two or more plane *Specula* duly posited; in which it is principally to be observed, that the Distance of any Image from the Eye, is equal to a Ray propagated from the primary Radiant, thro' all the intermediate Reflections to the Eye.

L E M-

LEMMA.

A Ray of Light is inflected by a curve-Surface after the same Manner, as it wou'd be by a plane Surface, touching the Curve in the Point of Incidence. *Fig. 3.*

Let P Q be any curve Surface, (for here, in the Figure, as likewise in all Cases hereafter, the common Sections of the inflecting Surface, and the touching Plane, with the Plane of Inflection, are used for the inflecting Surface, and the touching Plane themselves: because every Inflection is perform'd in the Plane of Inflection, as is demonstrated in *Axiom 2.* And for the same Reason in the Room of a solid Figure, we make use of a plane one described in the Plane of Inflection; which saves us a great many Words) upon which the Ray A B falls at B. Now since the Ray is of a Thickness not considerable, the Particle, in the Curve Surface B, upon which it falls, will be extremely small. But the Inclination or Direction of the Curve Surface at B, is the same with the Inclination of the Plane D E touching it in that Point: Wherefore the Inflection which depends upon the Direction, is likewise the same, whether it be conceiv'd, as occasion'd by the Particle B in the Curve Surface P B Q, or in the
C plane

plane one D B E. For as to the Inflection of the Ray A B, it matters not how the rest of the Surface is bent, if the Inclination of the small Particle B, upon which it falls, does but remain the same.

It wou'd be easy to demonstrate the same Thing, as the Antients did, from hence, that the Angle of Contact D B P, or E B Q, is less than any rectilineal one; and that a Plane may be found so inclined to the Plane D B E, that the Difference between the Inflections that are made by them both, shall be less than any given Inflection. For from hence it will follow, that the Inflection of the Ray A B, made at the Particle B of the Curve Surface, is no way different from that which is made at the Particle B of the plane Surface D B E, touching the Curve Surface in B.

PROP. III. PROB. II.

TO find the Focus of Parallel Rays falling upon a given Spherical Speculum, (or to find the Image of a vastly distant radiant Point) with respect to an Eye placed in the Axis of the Speculum, which is parallel to the incident Rays.

Thro' A the Centre of the *Speculum*, draw the right Line A B parallel to the incident Rays, meeting the *Speculum* in B. Bissect A B in C, I say C is the *Focus* required.

In this, as well as in all the following Propositions, we suppose the Point D to be extremely near the Point B. And this is necessary, in order that the reflected Ray belonging to the incident one E D, may meet with the Eye, which, by Supposition, is placed in A B, or the same produced: For the reflected Rays belonging to those that fall more remote from the Point B, do, after their diverging from their *Focus*, pass beyond the Pupil of the Eye; and consequently contribute nothing towards seeing the Image. Besides, of those Rays that enter the Pupil of the Eye, they that fall most directly, or nearest the Middle of the Pupil; (that is, that are reflected from those Points that are nearest to the Point B,) conduce more towards seeing the Image, than those that enter the Eye near the Extremes of the Pupil: Because those that fall most directly, and close to one another, move the Sense more forcibly, than those that fall more oblique and scatter'd. For which Reasons, we need have respect only to those that fall nearest to the *Vertex* B, which holds good in all the Propositions following.

Fig. 4.

Let ED be one of the parallel incident Rays. Draw AD , CD , and produce them. Because the Point D almost coincides with B , CD will be nearly equal to CB : But by Construction, CA is equal to CB ; therefore CA , CD are equal; and therefore the Angle CAD , CDA are likewise equal. But the Angle CAD is equal to EDA , which is the Angle of Incidence; for the right Line AD is perpendicular to the Surface of the Sphere. Whence the Angle CDA is equal to the Angle of Incidence of the Ray ED . Therefore, by *Corol. 3. Theor. 1.* DC is the reflected Ray belonging to the incident one ED . Moreover, the Angles at the *Vertex*, EDA , $eD\phi$, and CDA , $ND\phi$ being equal, the reflected Ray ND , belonging to the Ray eD parallel to BA , and falling upon the convex Surface, will, if produced backward, go to C . And what is demonstrated of any one Ray ED taken at pleasure, is true of all the rest in the same Circumstances. Wherefore, such Rays as are parallel to AB , and which, when reflected, conduce to Vision, if they fall upon a Concave Sphere, are, after Reflection, collected in C : and from thence again diverging, make the image to be seen in that Place, by an Eye placed in the *Axis*. The reflected Ray belonging to

to the very *Axis* it self does also, as it were, diverge from the Point *C*, in the Middle between *B* and *A* : For the same Thing happens to it, as to any other Ray reflected from *B D*, that cuts the right Line *A B* in a Point equally distant from the Centre *A*, and the Point of Incidence. But Rays that fall upon the Convex Side, do after Reflection, diverge from *C*, and make the Image appear in the Point *C*, (or which is all one, in a Point whose Distance from *C* is less than any given one) to an Eye placed in *A B* produced. *Q. E. D.*

COROL.

Hence, and from *Corol. 2. Theor. I.* it follows, that Rays diverging from *C*, and reflected from the Concave Surface, or converging towards *C*, and reflected from the Convex Surface, will be parallel to the right Line *A B*, joining the Centre of the Sphere, and the Point *C*.

SCHOLIUM. *Fig. 5.*

If with the Vertex *B*, the *Axis* *B G*, and a Parameter equal to the right Line *F B*, a Parabola be described, it will be the *least* of all that can be circumscribed about the given Circle *F B* ; or the Circle *F B* will be the *greatest* of all that can be inscribed with-

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within that Parabola, by *Corol. 1. Prop. XX. lib. 1. Vincentii Viviani de Maximis & Minimis*. And this Circle and Parabola will, at the Point B, *have the same Degree of Curvity*, (to use KEEPLER'S Words, in *Cap. VII. Prop. XX. Paralipom. in Vitellionem*) and are there most intimately united. For as the Contact of Lines is equivalent to two Intersections, and is really no way different from two Intersections infinitely near to one another, as Mathematicians know very well, and for the same Reason Surfaces that mutually touch one another, have the same Power in inflecting Rays that fall upon the Place of Contact, and in producing other Physical Effects, as is shewn in the foregoing *Lemma*: So this more intimate Union, and which is equivalent to four Intersections, and consequently the most intimate that can be between the Circle and conick Section, (as the Contact between this last and a right Line is of all the most intimate) this, I say, will, in Physical Effects, that depend upon Surfaces generated by the Revolutions of these Lines, produce a farther Equipollency. For as Rays that are parallel to G B, are by the concave Parabolick Conoid B D, reflected exactly to its *Focus C*, which is distant from the Vertex B, by a fourth Part of the Parameter of the generating Parabola; so the
same

same Rays reflected from the concave Sphere, are collected very nearly in the same Point; and the same Speculation holds in several other Physical Matters.

PROP. IV. PROBL. III.

THE Focus of *Diverging Rays* being given, whose Distance from a given concave Spherical Speculum, is greater than the fourth part of its Diameter; To find the Focus of these same Rays after their Reflection from the forementioned Speculum, with respect to an Eye placed in its Axis.

Fig. 6.

Through A the Centre of the Sphere, and the given Focus E, draw a right Line, meeting the Spherical Surface in the *Vertex* B; this I call *the Axis of Radiation*; and suppose the Eye to be placed in it some where or other. In this right Line take the Point C such, that BC may be to CA, as BE is to EA. I say C is the Focus required.

Let any Ray proceeding from E, fall upon the Concave Surface of the Sphere at the Point D, near enough the *Vertex* B, (for we have nothing to do with those that fall more remote, because after Reflection they affect not the Eye placed in EB, as has before been shown) in which Case ED will be nearly equal to EB, and may in

Physical

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Physical Matters be taken for it. Draw $A D$, $C D$. After the same manner $C D$ is equal to $C B$; and from whence by Construction $C D$ will be to $C A$, as $D E$ to $E A$. And inverting the Proportion, $C D$ is to $D E$, as $C A$ to $E A$. Wherefore by *Euc. Elem.* VI. 3. the Angles $A D E$, $A D C$ are equal; but $A D E$ is the Angle of Incidence of the Ray $E D$, and consequently $D C$ (by *Corol.* 3. *Theor.* I.) is the Ray $E D$ reflected from the concave *Speculum*. And since the Ray $E D$ is taken at Pleasure, it is plain the *Focus* of all the Rays diverging from E , after Reflection, will be in C , with Respect to an Eye placed in the *Axis* $E B$. *Q. E. D.*

C O R O L.

Hence likewise will be given the *Focus* C , of Rays $e D$ converging, in the forementioned Circumstances, towards the given *Focus* E , and reflected from a given convex Spherical *Speculum*.

SCHOLIUM. *Fig.* 7.

Rays diverging from E , are reflected from the Concave Sphere $B D$ converging to C , for this reason; because the Circle $B D$ described on the Centre A (by whose Rotation the Sphere is generated) has the same Degree of Curvity with an *Ellipsis*, described upon any *Foci* C and E (found
as

as in the foregoing Prop.) through B, and generating an oblong Spheroid, by which, as is commonly known, Rays diverging from one of the *Foci*, are from the concave Surface reflected, converging towards the other of them.

Draw the lesser *Semi-axis* H K, join the right Lines C K, E K, each of which are equal to the *Semi-axis* B H. From the Centre H upon C K, let fall the perpendicular H L.

It is plain L K is half the *Latus Rectum* belonging to the *Axis* B G, because it is a Third proportional to the right Lines C K and H K. (Which is also true, if from H the Point of Intersection of the greater *Axis*, with the right Line K H bisecting the Angle comprehended by the right Lines drawn from the *Foci* to any Point of the Curve, a right Line be let fall perpendicular upon either of the foresaid right Lines.) And consequently C L is half the difference between the greater *Axis* and its *Latus Rectum*; and is also a Third proportional to the right Lines, K C, C H, that is, to the greater *Semi-axis*, and half the distance of the *Foci*. Again, (by the Construction in the preceeding Prop.) B C is to C A, as B E to E A; from whence by changing and inverting the Proportion, B E will be to B C (E G) as A E to A C. Therefore
 $BE + EG \quad BE - BC :: AE + AC.$

$AE - AC$, that is, BG is to CE , as CE to $AE - AC$. And taking the halves of them, BH is to HC , as HC to HA : That is, HA is a Third proportional to the greater *Semi-axis*, and half the Distance of the *Foci*; whence it is equal to half the difference between the greater *Ax-is* and its *Latus Rectum*. Wherefore AB is half the *Latus Rectum*. And consequently the Circle BD is (by *Corol. 1. Prop. XX. lib. 1. Vinc. Viviani*) the greatest of all that can touch the Ellipsis BG on the inside at B . Wherefore (as is shown in the *Scholium* of the *preceeding Prop.*) it will in B have the same Curvity with the forementioned Ellipsis. From whence it comes to pass, that a Sphere thence generated, performs the same Thing in reflecting very nearly, which a Spheroid generated by the Ellipsis does exactly.

PROP. V. PROBL. IV.

THE Focus of diverging Rays being given, whose Distance from a given Concave Speculum, is less than the Fourth Part of the Diameter of the Speculum; To find their Focus after Reflection from the forementioned Speculum, with respect to an Eye placed in the Axis. Fig. 8.

Through

Through A, the Centre of the Sphere, and the given *Focus* E, draw a right Line, meeting the Spherical Surface in B. In this take the Point C such, that B C may be to C A, as B E to E A. I say C is the *Focus* required.

Let E D be any one of the incident Rays, draw A D, C D, and produce them; draw likewise the right Line, E R, parallel to the right Line C D.

Since the Arch B D is extremely small, E D, E B, and C D, C B will be equal. Wherefore E D is to E A, as C D to C A; that is, because (D C, E R are parallel) as E R to E A; therefore the right Lines, E R, E D, and consequently the Angles, E R D, E D R are equal. But E D R is the Angle of Incidence of the Ray E D, and E R D is equal to its alternate N D A, wherefore (by *Corol. 3. Theor. 1.*) D N is the reflected Ray belonging to the incident one E D. And since E D is taken at pleasure, it is plain that all the Rays proceeding from E, after they are reflected from the Concave Spherical Surface, will, if they be produced backwards, meet in C, or will have their *Focus* in C. *Q. E. D.*

COROL.

Hence, and from *Corol. 2. Theor. 1.* may be found the *Focus* E, of the Rays N D, converging towards a given *Focus* C, and reflected from a given concave spherical Surface B D.

PROP. VI. PROB. V.

THE *Focus* of *Diverging Rays* being given; To find their *Focus* after *Reflection* from a given *Convex spherical Speculum*, with respect to an *Eye* placed in the *Axis*. *Fig. 9.*

Thro' A, the Centre of the Sphere, and the given *Focus* E, draw a right Line meeting the spherical Surface in B: In this take the Point C in such manner, that A C may be to C B, as A E to E B. I say C is the *Focus* required.

Let E D be any incident Ray proceeding from E, draw C D and produce it. To this thro' E, draw E R parallel, meeting the right Line A D, produced in R. The Arch B D being evanescent, for the Reasons already given, C D will be equal to C B,
and

and ED to EB ; and therefore AC is to CD , as AE to ED , but because of the equi-angular Triangles ACD , AER , AC is to CD , as AE to ER . Therefore AE is to ED , as AE to ER . Wherefore ER is equal to ED . Therefore the Angle ERD , or its equal NDR , is equal to the Angle EDR , that is, to the Angle of Incidence of the Ray ED . From whence it follows, that DN is the reflected Ray of the incident ED . And since ED is taken any how, it is plain, all the Rays diverging from the *Focus* E , and entering the Eye, whose Position is given, will, after their Reflection from the convex spherical Surface BD , diverge from the *Focus* C . *Q. E. D.*

COROL.

Hence may be found the *Focus* E , towards which the Rays ND (converging before Reflection towards a given *Focus* C , whose Distance from the *Speculum* is less than the Fourth Part of its Diameter,) do, after their Reflection from a given convex *Speculum*, converge.

SCHOLIUM. Fig. 10.

It had been easy to have explained and demonstrated these two last Propositions,
and

and their Corollaries, in the same Words, and with different Figures, but this would have bred Confusion to Beginners. However, one *Scholium* will serve for both, to show that the Circle B D has the same Degree of Curvity, with the Hyperbola described upon the *Foci* C and E, through the Point B, by the Rotation of which Curve round its *Axis*, is generated the Surface of an Hyperbolick Conoid, performing exactly the proposed Reflection.

Draw the second Diameter K M. To the joined right Line B K, erect K L perpendicular, meeting the *Axis* G B in L, and H L will be a Third proportional to the right Lines, B H and H K, and consequently equal to half the *latus Rectum* belonging to the *Axis* B G. And since, from the Nature of the *Foci*, the Rectangle G C B is equal to the Square of the right Line K H; adding to both the Square of B H; the Squares of the right Lines B K and H C will be equal: Wherefore B K, H C are equal. And because of the Rectangular Triangle B K L, B L, or the Sum of half the Transverse *Axis*, and half the *latus Rectum*, is a third proportional to the right Lines H B, B K; or to the right Lines H B, H C; That is, to half the Transverse *Axis*, and half the Distance of the *Foci*. Again (by the Construction in the two preceeding Prop.) B C is to C A, as B E
to

to E A. Wherefore B E is to E G, as A E to A C. Therefore $BE - EG :: AE - AC$. $BE + EG :: AE + AC$, that is, B G is to C E, as C E to A E + A C. And taking the halves of them, B H is to C H, as C H is to H A: That is, H A is a third proportional to half the Transverse *Axis*, and half the Distance of the *Foci*. Therefore H A is equal to B L, or to the Sum of half the Transverse *Axis*, and half the *latus Rectum*. Wherefore A B is half the *latus Rectum*, and consequently the Circle B D is. (by *Corol. 1. Prop. 20. lib. 1. Vinc. Viviani*) the greatest of all that can touch the Hyperbola B G on the inside at B. Therefore, as has been shown already, it is equally curve in B with the Hyperbola. And hence it is, that the Surface of the Sphere, generated by the Circle, performs *very nearly* the same Thing in reflecting, which the Surface of the Hyperbolick Conoid does exactly: That is, it changes the *Focus* of the Rays E, into C; or the *Focus* C into E, as has been shown in *Prop. V. and VI.*

PROP. VII. PROB. VI.

THE Focus of Rays falling upon a given spherical Speculum being given; To find the Focus of the same Rays after Reflection, with respect to an Eye (even any where out of the Axis) given in Position. Fig. 2.

Hitherto we have supposed the Eye placed in the *Axis* of Radiation; that is, in a right Line drawn through the radiant Point, and the Center of the reflecting Sphere, both because most optical Instruments are made after this Manner, and because the Image seen by the Eye so placed, is much more lively and distinct than any other, because it is formed by Rays least scattered, and most exactly reflected, and upon this Account challenges to it self alone the Title of an Image.

But that it may appear that the Method before used in constructing Physical Problems, and demonstrating them when constructed, is more universal, and sufficient to determine the Image, seen by the Eye, however placed: Let the reflecting Sphere be signified by its greatest Circle BD, whose Plane passes through the radiant Point E, and O the Center of the Eye. It is required to find the Image of the Point E made
by

made by the reflected Rays, with respect to an Eye placed at O.

It is plain in the first Place, that the Image will be found somewhere in the reflected Ray passing thro' O. To find that Ray, this Problem must be solved, by which having two Points E and O, and (in the same Plane) the reflecting Circle given; it is required to find such a Point in the Circumference of the Circle, that a Ray falling upon it from either of the given Points shall be reflected to the other of them. And this is *Prop. XXXIX. Lib. V. Opticæ Albazeni*, to which he has premised 7 or 8 *Lemmata*, and is now commonly called *Albazen's Problem*. The Problem is in its Nature *solid*, and not to be constructed universally without the Intersection of a Conick Section with the given Circle. The Construction of this Problem has been published by several eminent Geometers, *Barrow*, *Slusius*, &c. but most elegantly by the most noble *C. Huygens* in the *Philosophical Transactions*, No. 98. We proceed therefore to determine exactly the Point it self C, where the Image is seen in the right Line O B, (drawn as those famous Men have directed).

Produce E B, O B, till they again meet the Circle in P and R. Bisect the Right Lines B R, B P in S and A, and divide S B in C so, that S C may be to C B, as

E

A E

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A E to E B. I say the Image by the Eye in O will be seen in C; and not in the Meeting of the reflected Ray D N with the right Line E Q, joining the radiant Point and the Centre of the Sphere, as *Euclid* in *Theor.* 17. and 18. *Catoptrica*, and others would have it.

Let the Ray E D fall upon the Point D, very near the Point B: For only those Rays, that fall after this Manner, do, after their Reflection, enter the Pupil of the Eye placed at O. Those that fall at a greater Distance, after Reflection, pass beside the Eye, and conduce nothing at all to Vision. Wherefore in investigating the Point C, where the Image is formed, D must be supposed to coincide with B: In which Case $ED = EB$, $CD = CB$, $AD = AB$, and $SD = SB$. Moreover, because O B is the reflected Ray of E B, the Angles A B Q, S B Q will be equal; and the Point D approaching near to B, and at last coinciding with it, the Angles A D Q, S D Q coinciding with the former A B Q, and S B Q (at least being very little different from them) will in that Case be likewise equal. But farther, the Point D approaching to B, the ultimate Angles, or the small and evanescent ones, D A B, D Q B, D S B will also be equal: For the Circle drawn through Q, and the coinciding Points D and B; that is, described on the Diameter B Q, and

and consequently touching the Circle $BPQR$ on the inside, passes through the middle Points of the right Lines BP , BR , howsoever drawn from the Point B : And consequently the Angles DAB , DQB , DSB , in this Case, are in the same Segment of the Circle passing through the Points D , B , S , Q and A . These Things being laid down, which follow from the Coincidence of the Point D with B ; since, by Construction, AE is to EB , as SC to CB , AE will be to DE , as SC to CD . And since in Triangles, the Sides are as the Sines of the Angles subtended by them; the Sine of the Angle ADO , or MDA , will be to the Sine of the Angle DAB , as the Sine of the Angle CDS is to the Sine of the Angle DSB ; because therefore the Consequents are equal, the Antecedents will likewise be equal; and consequently the Angles to which they relate, namely ADM and CDS , are equal. But it has been shewn before, that the Angles ADQ , SDQ are equal: Therefore the Angles QDM , QDC , and those opposed to them at the *Vertex* LDE , LDN are equal. Wherefore ND is the reflected Ray of the Incident one ED . And the same may be shewn of the other Rays, that meet with the Eye at O ; namely, that their reflected Rays will diverge from the Point C . Wherefore to the Eye receiving

these Rays only, the Image of the radiant Point E, made by Reflection, will appear at C. *Q. E. D.*

COROL. I.

Hence it follows, that Rays converging towards the Point C, will, after their Reflection from a convex spherical *Speculum*, converge towards E, and there form the Image, for an Eye placed in the right Line B E, any where beyond E. As likewise that the Image of the radiant Point C, made by Reflection from a concave spherical *Speculum*, will, to an Eye placed in the right Line E B produced, as in A, appear in E.

COROL. 2.

But if the radiant Point E be vastly distant, then the *Ratio* of the right Line A E to E B becomes a *Ratio* of Equality, and by Construction S C is equal to C B: That is, the Image of the vastly distant radiant Point situated in A B produced, made by a convex or concave *Speculum*, with respect to an Eye placed any where in B R, or the same produced, will be in the middle Point of the right Line B S.

The Eye being placed in the *Axis* of Radiation; that is, the right Lines B A and
B S

BS coinciding with the right Line BQ, this same Construction serves, and degenerates into the Constructions of the four preceding Propositions.

But the spherical Surface being changed into a plane one, then this Construction will be changed into that of *Prob. I.* For because the right Lines AE, SC are in that Case infinite, but their Difference remains still finite, the *Ratio* of them will be a *Ratio* of Equality: Therefore EB and BC, which are proportional to them, are equal. From whence it is plain, that this one Construction of this *Prob. VI.* contains all the former ones, as being more simple; which is frequent in Geometry.

SCHOLIUM.

The fore-mentioned Properties do therefore belong to the Circle BDR, because it has the same Curvity at the Point B, with a conick Section described through B upon the *Foci* C and E; which, as is known, reflects the Rays diverging exactly from C. The Equality of Curvity in the foresaid Case is from thence manifest: That the Segment of the Diameter of the Section (produced if need be) cut by the equicurve Circle, is equal to the *Latus Rectum* of that Diameter: And *vice versa*.

PROP. VIII. THEOR. II.

THE Image of a radiant Plane Surface made by a Spherical Speculum, is also a Plane Surface. Fig. 12, 13.

Hitherto we have determined the Image of any radiant Point with respect to an Eye, placed, either in the same *Axis* of the *Speculum* with the radiant Point, or out of it. From whence it is easy to determine the Images of radiant Bodies, because made up of radiant Points. But to the more easy Determination of it, as likewise to many other Things belonging to Practice, the *Theorem* before us will not a little conduce: In which we make use of that Image of any Point, which is seen by an Eye placed in the same *Axis* with the radiant Point. For we speak not here of those Images, that are seen by the Eye in any other Position (which yet may be determined by the Help of the preceding *Prop.*) because they are only secondary and less considerable; especially since such an Image of a right Line has been already sufficiently considered by the famous *Barrow*, in *Lect. Opt.* XVI. and XVII.

Let any spherical *Speculum* be signified by its greatest Circle *B D*, whose Centre
is

is A ; and the radiant Plane by the right Line F E. From A upon F E, suppose A E drawn perpendicular, meeting the *Speculum* in the *Vertex* B. Find, by some one of the preceding *Prop.* applicable to the Case, the *Focus* C of those Rays after Reflection, whose *Focus* before Incidence was E ; through this draw the Plane C T parallel to F E. I say that the Image of the Plane F E made by the *Speculum*, will be placed in the Plane C T.

From any Point F of the radiant Plane, to A the Center of the *Speculum*, suppose the Ray F A produced, meeting the *Speculum* in D, and the Plane C T in T. The reflected Ray of the incident one E D, is, by Construction, C D ; but let B H be supposed the reflected Ray of F B, meeting the Ray F D in H.

Because the Angle F A E is, by Supposition, small, the Arch D B will be also small, and almost degenerate into a small right Line. And the Circumference of a Circle described on the Diameter B F, will pass through the Points D and E ; because of the right Angles F E B, F D B. Whence the Angles B F D, B E D, in the same Segment, are equal. And since the Angles opposed at the *Vertex* A are equal, or the same, F B A, E D A will be likewise equal : Therefore the Angles A B H, A D C, that are by *Prop.* I. equal to these, are likewise

wise equal. Whence the Triangles $A B H$, $A D C$ are equiangular, and consequently similar. Wherefore $A B$ is to $A H$, as $A D$ to $A C$; and since $A B$ is equal to $A D$, $A H$ will be equal to $A C$. But because of the Smallness of the Angle $C A T$, $A T$ is reckoned equal to $A C$; therefore $A H$ is to be reckoned equal to $A T$. But H is the Image of the Point F in the radiant Plane; whence its Position is in the Plane $C T$. The same may after the like manner be shewn of any other Point in the Plane $F E$: From whence the Proposition is manifest.

But if the Object exposed to the *Speculum* be vastly distant, the Images of each of the radiant Points will, by *Prop. III.* be in the middle Points of the Semidiameters of the *Speculum* tending towards them: That is, the Image of a distant Object will form a spherical Surface, concentric with the *Speculum*. But because the radiant Body, by Supposition, is seen under a very small Angle, so small a Portion of the spherical Surface as is possessed by its Image, scarce differs from the plane Surface to which $A B$ is perpendicular. Wherefore the Proposition holds in all Cases: For the Demonstration takes Place in any other Case, as well as in those two expressed in the Figures.

COROL.

If the Angle EAF be too great, the right Line AH will be sensibly less than AT : whence the Image of an Object seen under too large an Angle, from the Center of the *Speculum*, made by a concave *Speculum*, will be sensibly concave; and that which is made by a convex *Speculum* will appear convex.

SCHOLIUM.

Concerning the Images of radiant Surfaces, made by spherical Surfaces, it will not be improper to take notice of some few Things. 1. If the Object exposed to the concave *Speculum* be distant from it more than by its Semidiameter, the Image of it, CT , will be distant from the *Speculum* more than by a fourth part of its Diameter, but less than by its Semidiameter; that is, it will appear hanging in the Air between the Object and *Speculum*, and likewise inverted; that is, the upper Parts will appear undermost, and the right on the left. If the Object recedes from the *Speculum*, the Image will approach to the *Speculum*; but if the Object approaches to the *Speculum*, the Image will recede from it, till meeting at last with the Object, it will coincide with it in the Center. After the

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same manner the Image of the Radiant CT will be FE , whose Properties are evident by what has been just now said. But if the Distance BE be so far encreased that the Object be vastly distant, the Distance of the Image CT from the *Speculum* will be equal to a fourth part of its Diameter. But if the Sun be the radiant Body, in the room of its Image will be excited a Burning (if the *Speculum* be considerably larger than the Image of the Sun) because of the Sun's Rays being closer compacted in that Place. But if a lucid Body be placed in the Middle, betwixt the *Vertex* and Center of the *Speculum*, its Image made by the *Speculum* at a vast Distance will enlighten Objects vastly distant.

2. If CB , the Distance of the Radiant CT from the concave *Speculum* B , be less than a fourth part of its Diameter, its Image will be EF (by *Prop.* IV and VIII) placed beyond the *Speculum*, and erect as in a plane *Speculum*, and which approaches to the *Speculum* as the Object approaches to it; and so on the contrary. After the same manner the Image of the Radiant FE , made by a convex *Speculum*, will be CT , whose Properties, from what has been said, are easily detected: For as the Radiant approaches to the *Speculum*, the Image likewise approaches to it; but when that recedes to a Distance even infinite, the Image will

will stop at the middle Point between the *Speculum* and its Center.

3. Suppose the Rays of each Point of the Radiant were so inflected as to be about forming the Image *CT*, but are hindered from it by the Intervention of the convex *Speculum* *BD*. From what has been said above, it is plain that these Rays, after Reflexion from the *Speculum*, will make the Image *FE*; whose Properties, Situation, and Figure, with respect to the Object *CT* (if it may be called an Object) are easily found by the foregoing Rules.

In the preceding, and all other Cases, the *Ratio* of the Image to the Object is given, because they appear under the same or equal Angles from the Center of the *Speculum*. But this will be more plain by the following *Theorem*.

PROP. IX. THEOR. III.

THE Radiant and its Image made by the *Speculum*, are seen from the Vertex of the *Speculum* under equal Angles.
Fig. 14, 15.

From *A*, the Center of the *Speculum*, upon the Radiant *HF*, let fall the Perpendicular *AE* (which, by *Prop. VIII.* will
F 2 be

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be likewise perpendicular to the Image fb meeting the *Speculum* in the *Vertex* B . Join the right Lines BF , BH , Bf , Bb . I say the Angles HBF , bBf , are equal. Join Ff ; this will pass through the Center A , because by Supposition the Image of any Point is placed in the *Axis* of Radiation. After the same manner the Points HA and b are placed in the same right Line. From the Nature of the Image, BE is to EA , as BC to CA : therefore BE is to BC , as EA to CA . But because of the equiangular Triangles, AEF , ACf , EA is to CA , as EF to Cf . Wherefore BE is to EF , as BC to Cf . And since the Angles BEF , BCf are right, the Angles EBF , CBf will be equal. After the same manner EBH , CBb are equal: Whence HBF , bBf are equal. *Q. E. D.*

C O R O L.

The radiant Line FEH , and its Image fCb , are to one another as their Distances from the *Vertex*, BE , BC . And if the Radiant be a Surface, it will be to its Image in a duplicate Proportion of those same Distances. From whence, if the Distance of the Radiant and its Image from the *Speculum* be given, the Proportion of the Image to the Radiant is also given. In like manner;

manner, if the Image of the Radiant C made by several *Specula* be given, we shall have the Proportion of the primary Radiant to the last Image. It follows farther, that the Line FH , and its Image fb , will be cut in the same Proportion by the right Line BE , joining the *Vertex* and *Center*; or that FE is to EH , as fC to Cb .

PROP. X. PROB. VII.

TO find such a Position of the radiant Body, that its Image made by the Speculum may be equal to any given Figure similar to the Radiant: Or, which comes to the same thing, that the Radiant and its Image made by the Speculum may be in a given Ratio. Fig. 16.

Let the *Speculum* BD be given, whose Semidiameter is BA ; and let the given Ratio be BA to AM . If the Radiant be a Line, bisect BM in E , and by some of the preceding Propositions, find the *Focus* C , corresponding to the *Focus* E . I say, C is the Place of the radiant Line; because, from the Relation of the Points C and E , BC is to CA , as BE to EA ; BC will be to $BC + CA$, as BE to $BE + EA$: that is, because BE is equal to EM , BC will

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will be to BA , as BE to AM ; and inverting the Proportion, BC is to BE , as BA to AM . But (by *Corol. Prop. IX*) BC is to BE , as the radiant Line placed in C , to which BC is perpendicular, is to its Image in E : Therefore the radiant Line in C is to its Image made by the given *Speculum* BD at E , as BA to AM ; that is, in the given *Ratio*. *Q. E. D.*

But if the Radiant be a Surface, in the room of AM in the preceding Construction you must take a right Line, to which BA is in a *Ratio* subduplicate of that which it bears to AM .





THE
ELEMENTS
OF
DIOPTRICS.



Atoptrics being finished, we proceed to Dioptrics; by the Assistance of which we are still furnished with more Instruments, and such as are fitter for Use, and admitted to a nearer View of the Secrets of Nature. For Glass is easier brought to a due Figure than Metal, and preserves it longer: Nor does a *Lens* suffer so much Loss in its Polish, from any Injuries of the Air, as a *Speculum* does. And it is difficult to make a *Speculum* that shall reflect such strong and close Rays as a Glass *Lens* transmits.

But, besides these physical Difficulties, the Causes of which are not to be sought from
from

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from Dioptrics, a *Lens* is preferable in Practice to a *Speculum*, for the following Reason: Because a Fault in a *Speculum* produces an Error in the reflected Ray six times greater than an equal Fault in a *Lens*, when the Ray passes out of Air into Glass; and four times greater, when the Ray passes out of Glass into Air: For, by *Theor.* I. the Error of Incidence produces an Error twice as great in the reflected Ray, but near thrice less in a refracted Ray out of Air into Glass; and twice less in a refracted Ray out of Glass into Air; as will appear at *Schol.* 2. *Prop.* XI.

In Dioptrics, as before in Catoptrics, we suppose the Eye placed in the *Axis* of Radiation; because this Position, where only *Lenses* are used, is easiest obtained: But where *Specula* and *Lenses* are mixed together, that *Axis* (by *Corol.* 1. *Prob.* I.) may be inclined in any given Angle.

PROP. XI. THEOR. IV.

A Ray of *Light*, at a plane Surface of a Medium of different *Density*, is so refracted, that the Media remaining the same, the right Sine of the Angle of Incidence will, in all Inclinations, bear the same Proportion to the right Sine of the refracted Angle. Fig. 17.

It

It is sufficient for a Mathematician to have proved this *Theorem* by Experiment; so that from this laid down he may demonstrate the Properties of a given Figure in refracting, or investigate a Figure whose Laws of refracting are given: Yet, for the same Reasons that induced us to demonstrate a like Catoptrical *Theorem*, we shall give the Demonstration of this.

For this Purpose, different Persons have used different, and quite contrary, *Postulata*. *Cartes* taking up the same *Theorem* (though in other Cases he affirms the Propagation of Light to be instantaneous) would have it in his *Dioptrics*, that a Ray of Light is carried with a greater Celerity through a denser *Medium*, as Water or Glass, than through one less dense, as Air. This appeared too gross to *Barrow*, *Fermat*, and others, who went into a contrary and more probable Opinion. Moreover, *Barrow* and *Maignanus* believed, that some certain Thickness of the Ray of Light, however small (which *Cartes* neglected as inconsiderable) was necessary to demonstrate this *Theorem*. We have followed the Method of Geometers, and composed a Demonstration far enough from any Sect, and depending only upon a very simple Property of Light, which we assumed in the beginning.

G

Let

Let the Light flowing from the Point A, and comprehended in the Plane of Inflection between A B and A D, fall upon the right Line B D, the common Section of the Plane of Inflection with the plane Surface of the *Medium* B C E D of different Density. (We may justly consider this by it self; because the Rays which compose it, and their refracted Rays, are, distinctly from the other circumfused Light, contained within the forementioned Plane by *Ax.* 2.) Let one of the extreme Rays A D be perpendicular to the right Line B D, whose refracted Ray D E (whatsoever be the Density of the *Medium* B E) will consequently go directly on in the same right Line with A D: But let the other A B, containing a certain Angle with A D, meet the right Line B D, which separates the *Media*, in B; through B draw K B F parallel to A E. Let the aforesaid Light, contained betwixt the right Lines A B, A D, be supposed to be propagated in the *Medium* D G, betwixt the right Lines D E and B G continued *in infinitum*: That is, let B G be the refracted Ray of A B, whether the Light passes out of a rarer *Medium* into one more dense, as in *Fig.* 17. or out of a denser one into a rarer, as in *Fig.* 18. I say, the Sine of the Angle A B K will, in any Inclination of the Ray, have the same Proportion to the Sine of the Angle G B F.

On

On the Center B, and with any Distance BG, describe an Arch of a Circle meeting the Ray BG in G; from which let fall GN perpendicular to AN, intersecting the right Line KF in F.

The Light which proceeding from A is diffused into the right Line BD, will, if BG be put for the Length of the refracted Ray, be diffused within the new *Medium* DG into the right Line GN: For the Spaces into which the forementioned Light is diffused, must be computed by these right Lines parallel to the right Line BD, because at BD it loses its former Degree of Diffusion, and after that enjoys a new one. The right Line BD which divides the *Media* remaining the same, suppose the *Medium* DG, in which AB is refracted into BG, were taken away, and another *Medium* DC of a different Density substituted in its Place, in which the Ray AB is refracted into BC, *Ex. gr.* meeting the Circle in C, through which draw the right Line CE parallel to BD, meeting AN in E, and KF in M. The right Line CE will be the Space into which the Light is diffused within the *Medium* DC, the Length of the refracted Ray BC continuing still the same. If from the right Lines GN, CE, into which the Light is diffused, you take away the right Lines FN, ME, which depend not at all upon

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the *Facilities* of the *Media*, that is, their *Readiness in diffusing Light* (for the Length and Position of the Ray AB remaining the same, the forementioned right Lines FN , ME remain the same and invariable, howsoever the *Facilities* of the *Media* DG , DC are augmented, diminished, or even annihilated) there will remain GF , CM for the genuine Effects of the *Media* DG , DC respectively. Therefore since all Things that respect the Refraction of the Ray AB from the *Medium* DK , to *Media* of different Densities DG and DC , excepting only the Densities of those *Media*, are made the same (namely, the *Medium* DK , out of which the Light passes the same, the Angle of Incidence ABK of the Ray AB the same, and the Length of the refracted Ray within the second *Medium* DG , or DC the same) the forementioned right Lines GF , CM will have the same Proportion with the *Facilities* of the *Media* DG , DC which produced them: For all Effects are proportional to their Causes.

Now if the *Medium* DC , in which the Ray AB is refracted into BC , be the same with the *Medium* DK , in which Case the refracted Ray BC (if it may be called so) of the Ray AB will be in a direct Line with the incident one AB ; GF will be yet to CM as the Facility of the *Medium*
 DG

DG to the Facility of the *Medium* DC, that is, by Construction, to the Facility of the *Medium* DK. Moreover, CM is the Sine of the Angle CBM, or the Angle ABK, which is the Angle of Incidence of the Ray AB; and GF is, to the same *Radius*, the Sine of the Angle GBF, that is, of the refracted Angle of the same Ray: Wherefore the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, whatsoever be the Inclination of the Ray AB to the refracting Plane (for the Angle DAB, to which ABK is equal, is taken at Pleasure) as the Facility of the *Medium* DK to the Facility of the *Medium* DG, inflecting the Ray AB into BG. But the *Media* being supposed the same, the Densities of the *Media*, and their *Facilities* arising from thence, and consequently the *Ratio* of these will remain the same: Therefore in every Inclination of the Ray the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction remains the same. Q. E. D.

This *Theorem* is thus shortly demonstrated in the Geometric Phrase. The Position of the incident Ray remaining the same, the refracted Ray remains the same, whatsoever be the Length of the incident Ray AB: Therefore no respect is to be had to its Length, or the right Line AB is to be taken for nothing. Therefore the right
Line

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Line FN , which bears a certain Proportion to AB , vanishes at the same time : Whence the right Line GN , into which the Light is diffused at a given Distance BG within the *Medium* DG , is changed into the only and determinate right Line GF ; which is consequently appointed by Nature for the Measure of the Facility of the *Medium* DG , to which it owes its Rise.
Q. E. D.

But if in the Passage of a Ray from a denser *Medium* into a rarer (*Fig. 18.*) its Inclination be such, that GF , which is to CM in the Proportion of the *Facilities* of the given *Media*, should exceed the *Semi-diameter* BG ; then the refracted Ray of AB will be nothing (after the manner of an impossible Case in a *Geometrical Problem.*) But that which should have been the refracted Ray will not enter the rarer *Medium* DG , but will be reflected from its Surface, according to the Law of *Theor. I.* in which Case the Law of this present *Theorem* (as being more universal) does nevertheless take place.

The converse of this Proposition is sufficiently manifest ; namely, that the refracted Ray of any Incident is truly assigned, when the Angle of Incidence is to the Angle comprehended by the assigned right Line, and another drawn perpendicular to the refracting Surface, in the Proportion of the
Faci-

Facilities of the given *Media*, which is the Measure of the Refraction between them: For the refracted Ray of the Incident proposed can be no other than that assigned.

The Invention of the preceding *Theorem*, which is the principal one in *Dioptrics*, is commonly attributed to *Cartesius*, tho' it was known to *Willebrord Snellius*, who was dead ten Years before *Cartes's Dioptrics* were published: For *Vossius*, in his Treatise *De Natura & Proprietate Lucis*, published at *Amsterdam* in the Year 1662, Page 36, tells us, that it appeared from *Snellius's* Papers, which himself had seen, that he had found out, that the Proportion between the Secants of the Angles, which are the Complements of the Angle of Incidence, and the Angle of Refraction to right ones, is constantly the same: But it is known that the Secants of Angles are reciprocally, as the Sines of their Complements to right ones; because the *Semidiameter* is a mean Geometric Proportional between them.

Since we have happened to speak of Secants, it is worth the taking notice how near the inquisitive *KEPLER* was towards finding out this *Theorem*; who, at *Prop. V. and VI. Paralipom. in Vitellionem*, lays down these Secants for the respective Measure of Refractions.

C O R O L.

C O R O L. I.

If B G be the refracted Ray of the Incident A B, every thing else remaining the same, B A will be the refracted Ray of the Incident G B: For the *Facilities* of the *Media*, by which the Sines of the Angles are governed, remain the same.

C O R O L. 2.

From the Demonstration of this *Theorem* it follows, that if the *Ratio* of A to B measure the Refraction between the *Media* A and B, and the *Ratio* of A to C the Refraction between the *Media* A and C; the *Ratio* of B to C will be the Measure of the Refraction between the *Media* B and C. Which is also manifest, by supposing the Thickness of the common *Medium* A, between the parallel Planes of the other two, to vanish: For by this means, the Angles remaining the same, that Part of the Ray, propagated through the three *Media*, which is in the *Medium* A, vanishes.

C O R O L. 3.

Hence it likewise follows, that if the Position of the Ray A B remaining the
same,

same, the Position of the inflecting right Line BD be changed by an Angle, suppose E , the Position of the refracted Ray BG will also be changed by an Angle e , which is to E as the Difference of I and R to I ; (the *Ratio* of the Quantities I and R being supposed the same with the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction; and the Angles so small, that they may be looked upon to have the same Proportion with their Sines) and the Angles e and E are situated on the same Sides of the right Lines BG and BD respectively, when I exceeds R ; but on contrary Sides when R exceeds I . After the same manner, from this Proposition, may we judge of the Change the Angle GBF will undergo, when the Angle ABK is changed by any other Cause whatever.

SCHOLIUM I.

The aforefaid Law of Refraction is confirmed from the Wisdom of Nature, always acting by the most easy and expeditious Methods; which we before found in *Catoptrics* to be a legitimate *Axiom*; namely, to the Light proceeding from A to G , such a Point of Incidence B is assigned, that it may perform its Journey ABG in the *least Time* possible. Which

H
Point

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Point of Incidence may be found by the Help of any Method (*Fermat's* for Instance) which determines a *Maximum* & *Minimum*, but most easily after the Manner following.

P R O B.

TWO Points being given in Media of different Densities, and the plane Surface dividing the Media, whose Densities are likewise given, being given in Position: To find such a Point in the foresaid Surface, that a moveable Body proceeding from one of the given Points, through the Point sought to the other of them, may take up the least Time in its Journey. Plate II. Fig. I.

Let the given Points be A and G, and let MN be the Surface dividing the *Media* of different Densities, or rather the common Section of the Plane of Inflexion continued through A and G, with the foresaid Surface dividing the *Media*: For (by *Ax. 2.*) the Point sought will be found in that. Let the right Lines I and R express the Velocities of Light in the *Media* DK, CF respectively.

Because the Velocity of a moving Body being given, the Time is as the Space run; and the Space being given, the Time is as

as the Velocity reciprocally; if neither being given, the Time will be in a compound Proportion of the Velocity reciprocally, and the Space directly. Therefore the Time in which the right Line A B is run will be expressed by $R \times A B$, and the Time in which B G is run, by $I \times B G$; and consequently the Time in which the whole Journey A B G is performed, which is made up of both of them, will be expressed by the Quantity $R \times A B + I \times B G$; which must be the *Minimum*.

From the Position of the Points A and G, and of the right Line M N being given, the right Lines A D, G C, and D C, which we shall call a , c , and d respectively, are given in Magnitude. Call C B x . Whence $B D = d - x$, $A B = \sqrt{d^2 - 2dx + x^2 + a^2}$, and $B G = \sqrt{c^2 + x^2}$. Therefore $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 + x^2}$ must be a *Minimum*.

Now this is (as Mathematicians know very well) when the *Fluxion* of $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 + x^2} = 0$. But by the *Method* of that most celebrated Geometer Sir Isaac Newton, for finding the *Fluxions* of as many *Fluents* as are involved in any given Equation (to which also the famous Leibnit's *Differential Calculus* relates) which you may see at *Cap. XCV. Vol. II. Operum Mathematic. Wallisii*, The

H 2
Fluxion

Fluxion of $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 + x^2}$
 is $R \times \frac{-2dx + 2xx}{\sqrt{d^2 - 2dx + x^2 + a^2}} + I \times \frac{2xx}{\sqrt{c^2 + x^2}}$.

Therefore

$$\frac{2\sqrt{d^2 - 2dx + x^2 + a^2}}{2\sqrt{c^2 + x^2}} + \frac{I \times 2xx}{2\sqrt{c^2 + x^2}}$$

 making this equal to nothing, we have

$$\frac{I \times x}{\sqrt{c^2 + x^2}} = \frac{R \times d - x}{\sqrt{d^2 - 2dx + x^2 + a^2}}.$$

And substituting, in the room of their Va-
 lours, the right Lines themselves expressed
 $I \times CB \quad R \times BD.$

in the Scheme, it becomes $\frac{BG}{AB} = \frac{AB}{AB}$

And if $BG = AB$, then $I \times CB = R \times BD$, or
 $B.D. : B.C. :: I. R.$

Therefore if the right Line DC be di-
 vided in B , so that DB may be to BC in
 the given *Ratio* of I to R , the Light will
 perform its Journey from A to G (or
 backward from G to A) in the shortest or
 least Time possible, by going along ABG .
 But DB and BC are the Sines of the
 Angles BAD and BGC ; that is, of the
 Angles ABK , GBF ; namely, of the An-
 gle of Incidence, and the Angle of Refrac-
 tion. Wherefore, that the Time of the
 Passage from A to G (or from G to A)
 may be the *least*, the Ray must so fall,
 that the Sine of the Angle of Incidence may
 be to the Sine of the Angle of Refraction
 in the *Ratio* of I to R ; that is, in the
Ratio of the Velocities of Light in the fore-
 said

said *Media*, or (as is shewn before in the Demonstration of *Prop. XI.*) in the *Ratio* of the *Facilities* of the same *Media*, upon which the *Velocities* of the Body moving through them depend.

Since there are in the Plane dividing the *Media* an infinite Number of such Points (two of which are in the right Line M N) that the Light, in passing from A to G (or from G to A) through any of them, shall take up any given Time exceeding the *least*; there can be no Reason assigned, why it should pass through one of those Points rather than another: Therefore it will pass through none but the only and determinate one of its kind, B.

SCHOLIUM 2.

When a Ray of Light passes out of Air into Glass, we observe the Sine of the Angle of Incidence to be to the Sine of the Angle of Refraction, as 3 to 2 in round Numbers: And in passing out of Air into Water, I is to R (which Symbols we shall use for the future, to express the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, which is the Measure of Refraction) as 4 to 3. And on the contrary, in the Passage of a Ray out of Glass into Air, I will be to R as 2 to 3, by *Corol. 1.* From whence the Reason is plain, why a Ray in Glass, striking
upon

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upon a Surface of Air, more obliquely than in an Angle of Incidence of about 42° Degrees, does not enter the Air, but is reflected from its Surface: For the right Line GF , which is sesquialteral of the Sine of 42° , exceeds the *Radius*; in which Case the Light shall be reflected, as has been observed above.

Moreover, because in the Passage of a Ray out of Air into Glass, I is sesquialteral or $1\frac{1}{2}$ of R , and in its Passage out of Air into Water, I is $1\frac{1}{3}$ of R ; in its Passage out of Water into Glass (by *Corol. 2.*) I will be $1\frac{1}{8}$ of R .

And from *Corol. 3* it follows, that the Error of the refracted Ray is but subtriple of the Error of the Surface of Glass by which it was occasioned, and on the same Side with it, when the Ray passes out of Air into Glass; but subduple of it, and on the contrary Side, when the Ray passes out of Glass into Air.

PROP. XII. THEOR. V.

LET BD be a Surface dividing Media of different *Densities*; upon which let a Ray fall proceeding from E , and be refracted into DN , the Surface dividing the Media remaining the same, but the Media

Media being transposed ; I say the refracted Ray DC of the Ray MD being placed in the same right Line with ED, will be in the same right Line with the former refracted Ray ND. Fig. 2.

Draw the right Line ADO perpendicular to the Surface BD in D.

Since DN is the refracted Ray of ED, (by the foregoing) the Sine of the Angle EDA will be to the Sine of the Angle NDO in the *Ratio* of I to R ; and if the *Media* be transposed, the *Ratio* of I to R will be the Measure of the Refraction out of the *Medium* BDM into the *Medium* BDE : That is, supposing MD to be the incident Ray, its refracted Ray will be such, that the Sine of the Angle MDO shall be to the Sine of the Angle ADC, as I to R : That is, by what has been just now shewn, as the Sine of the Angle ADE to the Sine of the Angle NDO. Therefore since the Angles EDA, MDO are equal, and consequently their Sines, the Sines of the Angles ADC, NDO will be likewise equal, and consequently the Angles themselves. Wherefore ND and DC lie in the same right Line. Q. E. D.

COROL.

If Rays diverging from E, after Refraction at the Surface BD, diverge from

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from the Point C, the *Media* being transposed, and the Surface dividing the *Media* remaining the same, Rays converging towards E will, after their Refraction at B D, converge towards the Point C.

Therefore all that can be demonstrated of Rays diverging, may equally be applied to converging ones. Wherefore for the future we shall only speak of Rays diverging, leaving those converging to be determined by this *Theorem*.

P R O P. XIII. P R O B. VIII.

FROM the Focus of diverging Rays being given, to find the Focus of the same Rays after their Refraction, at a plane Surface of a Medium of different Density, with respect to an Eye placed in the Axis. Fig. 3, 4.

Through E, the given *Focus* of diverging Rays, draw the right Line E B perpendicular to the plane Surface B D of a *Medium*, either denser or rarer; in this take the Point C such, that C B may be to E B, as I to R. I say C is the *Focus* sought.

Let fall any Ray E D. Through D draw A O parallel to E B. Join C D, and produce it to F.

If

If the Ray ED be near enough to the right Line EB (for we speak here only of those Rays that fall near the *Axis*, since these only do, after Refraction, affect the Eye placed, by Supposition, in the *Axis* of Radiation produced; those that fall more obliquely passing by the Pupil of the Eye, and conducing nothing at all towards discerning the Point E , as we have before observed in *Catoptrics*) ED will be nearly equal to EB , and CD to CB . Wherefore CD is to ED , as I to R : But CD is to ED , as the Sine of the Angle BED , or its equal EDA , to the Sine of the Angle BCD , or its equal ODF : Therefore the Sine of the Angle EDA is to the Sine of the Angle ODF , as I to R . But the Angle EDA is the Angle of Incidence of the Ray ED : Therefore ODF is the respective Angle of Refraction; that is, DF is the refracted Ray of the Incident ED . And the same may be shewn of any other Ray diverging from E ; whence C is the *Focus* required. *Q. E. I.*

COROL. I.

From hence, and from *Corol. 1. Prop. XI.* it follows, that the same Things being supposed as before, Rays in the *Medium* BDO converging towards C , will, after Refraction, converge towards E .

I

And

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And if the *Media* were transposed, it follows from hence, and from *Prop.* XII. that Rays in the *Medium* B D O converging towards E, will, after Refraction, converge towards the Point C; and parallel Rays will, after Refraction, at a plane Surface of any *Medium*, still remain parallel.

C O R O L. 2.

If B D A be Water, and B D O Air, C B will be to E B, as 3 to 4: From whence it happens, that Water appears a fourth part less deep than it really is. But if B D A be Glass, E B will be $1\frac{1}{2}$ of the right Line C B.

S C H O L I U M.

What is demonstrated of any incident Ray, namely, that its refracted Ray D C produced backwards, till it meets with E B, is to the Incident E D as I to R, is also true of the perpendicular Incident E B: For its refracted Ray B C produced backwards, though as to its Position it be the same with the Incident, yet as to its Length it is to it in the very same *Ratio* of I to R. And because this Ray passing through the Center of the Eye, and the others that are nearest it are the only ones
that

that affect the Sense ; from thence it is that, with respect to the Eye, C is the *Focus* of the Rays diverging from E, and refracted at B D. This Consideration extends likewise to what follows.

PROP. XIV. PROB. IX.

TO find the Focus of parallel Rays falling upon a spherical Surface of a Medium of different Density, after Refraction. Plate II. Fig. 5, 6.

Let the spherical Surface, whose Center is A, be expressed by the Circumference of the Circle B D ; through A draw the *Axis* A B parallel to the incident Rays, meeting the Circle in the *Vertex* B ; in which take the Point C such, that B C may be to A C, as I to R. I say C is the *Focus* required.

Let any Ray fall at D ; draw the right Lines A D, C D, and produce them. The Sine of the Angle B A D, or its equal E D O, is to the Sine of the Angle C D A (or its Complement to two right ones) as C D to A C, or (the Arch D B vanishing, for Reasons before often mentioned) as C B to A C ; that is, by Construction, as I to

I 2

R :

R: But $E D O$ is the Angle of Incidence of the Ray $E D$; therefore $D C$ is its refracted Ray. After the same manner it may be demonstrated, that any other of the parallel Rays will, after Refraction, pass through C : Therefore C is their *Focus*.
Q. E. D.

C O R O L L A R I E S.

Hence, and from *Corol. 1. Prop. XI.* and *Corol. Prop. XII.* it follows;

1. That Rays in Air parallel to the *Axis*, after Refraction at a *Spherico-convex* Surface of Glass, converge to a Point whose Distance from the *Vertex* is equal to three Semidiameters of the Sphere.

2. That Rays in Glass, diverging from a Point distant from a *Spherico-concave* Surface of Air by three Semidiameters, do, after Refraction, become parallel to a right Line drawn through the radiant Point and the Center of the Sphere.

3. That Rays in Glass parallel to the *Axis*, after Refraction at a convex spherical Surface of Air, diverge from a Point whose Distance from the *Vertex* is equal to the Diameter of the Sphere.

4. That Rays in Air, converging towards a Point distant beyond a concave Glass by the Diameter of the Sphere, after Refraction at the concave Surface of Glass, become

become parallel to a right Line drawn through the Center of the Sphere and the forementioned Point.

5. That Rays in Air parallel to the *Axis*, after Refraction at a concave Surface of Glafs, diverge from a Point at three Semidiameters Distance from the *Vertex*.

6. That Rays in Glafs, converging to a Point three Semidiameters distant beyond a *Spherico-convex* Surface of Air, after Refraction, become parallel to a right Line drawn through the Center of the Sphere and the forementioned Point.

7. That Rays in Glafs parallel to the *Axis*; after Refraction at a *Spherico-concave* Surface of Air, converge to a Point distant from the *Vertex* by a Diameter of the Sphere.

8. That Rays in Air, diverging from a Point at a Diameter's Distance from a Sphere of Glafs, after Refraction at a convex Surface of Glafs, become parallel to a right Line drawn through the forementioned Point and the Center of the Sphere.

SCHOLIUM. *Plate II. Fig. 7.*

The generating Circle of the Sphere B D has the same Degree of Curvity with an *Ellipsis* passing through B; by whose Rotation a *Spheroid* being made of a denser *Medium*, collects Rays in the ambient *Medium*

Medium parallel to the right Line A B, exactly at C: For we all know, that B G, the greater *Axis* of that *Ellipsis*, ought to be to E C, the Distance of the *Foci*, as I to R. Supposing then the same Construction with that at *Schol. Prop. IV.* B G is to E C as K C to H C; that is, as H C to C L; that is, as K C + H C to H C + C L, or to H C + C K — L K; that is, as B C to B C — half the *Latus Rectum*, belonging to the *Axis* B G (for L K has, at *Schol. Prop. IV.* been shewn equal to half the forementioned *Latus Rectum*) But by the Construction of this *Prop.* the Situation of the Point C is such, that B C is to A C as I to R; that is, as B G to E C: Therefore B C is to A C, as B C to B C — half the *Latus Rectum*, belonging to the greater *Axis*: Wherefore A C is equal to B C — half the forementioned *Latus Rectum*; and consequently A B is equal to half the *Latus Rectum*. Wherefore the Circle B D is equally curve in B with the *Ellipsis* B K G (by *Corol. 1. Prop. XX. Lib. 1. Vinc. Viviani de Maximis & Minimis.*) Whence it is, that a Sphere generated by the forementioned Circle performs in refracting very nearly the same thing which an oblong *Spheroid*, generated by the Rotation of an *Ellipsis*, does exactly; namely, that Rays in a rarer ambient *Medium*, parallel to the right Line B G, after Refraction

tion at its Surface, may converge to the Point C.

After the same manner, by the Help of *Schol. Prop. VI.* it may be shewn, that the Circle B D is at the *Vertex* equally curve with an *Hyperbola* passing through B, and generating a Conoid; which being of a rarer *Medium*, does so refract Rays in a denser ambient *Medium* parallel to the right Line A B, that they shall afterwards diverge from the Point C: Or, which returns to the same thing (by *Prop. XII.*) being of a denser *Medium*, does so refract Rays in that same denser *Medium*, parallel to the right Line A B, at their Entrance into the rarer ambient *Medium*, that they shall afterwards converge to the Point C.

PROP. XV. PROB. X.

THE Focus of Rays diverging and falling upon a given spherical Surface of different Density, to find the Focus of the same Rays after Refraction. Plate II. Fig. 8, 9, 10, 11, 12, 13, 14, 15.

Let the spherical Surface be expressed by the Circumference of its greatest Circle B D. Through the given *Focus* and the Center

Center of the Sphere draw the right Line EA meeting the Circumference in B ; in which take the Point C such, that the *Ratio* compounded of the *Ratio* of EA to AC , and of CB to BE , may be equal to the *Ratio* of I to R . I say C is the *Focus* required.

We shall make one Demonstration serve for all the eight principal Cases of this *Prop.* expressed by so many several Figures; the four first of which, suppose I greater than R , the others less, and which differ from one another, according as the given *Focus* is situated on this or that Side of the *Focus* of parallel Rays, or according as the refracting Surface is convex or concave.

Let fall any Ray at pleasure, ED proceeding from E . Join AD , DC ; and through C draw a right Line parallel to AD , meeting the right Line ED in H .

The Point D almost coinciding with B , CD and CB , as likewise ED and EB , are almost equal. Wherefore the *Ratio* of I to R is equal to the *Ratio* of EA to AC , and the *Ratio* of CD to ED together; that is (because of AD and CH parallel) to the *Ratio* of CD to ED , and the *Ratio* of ED to DH together, or to the *Ratio* of CD to DH : But CD is to DH as the Sine of the Angle DHC , or its Complement to two right ones; ADH ,
or

or EDO , is to the Sine of the Angle DCH , or of the Angle ADC , or in some Cases its Complement to two right ones. Moreover, the Angle EDO is the Angle of Incidence of the Ray ED : Therefore as I to R , so the Sine of the Angle of Incidence of the Ray ED to the Sine of the Angle ADC . Wherefore (by *Prop. XI.*) DC is the refracted Ray belonging to the incident one ED ; and since ED is taken at Pleasure, 'tis plain the *Focus* of all the Rays proceeding from E will be C . Q. E. I.

If the *Media* proposed were Air and Glass, and the Ray passed out of Air into Glass (as in the four first Figures) the *Focus* C will be very easily found; namely; if the third part of the right Line EA be to AB , as EC to BC : For trebling the Antecedents, EA is to AB (or AD) as $3 EC$ to BC (or DC): But EA is to AD , as the Sine of the Angle EDO to the Sine of the Angle DEA ; and $3 EC$ is to DC as thrice the Sine of the Angle EDC (or HDC) to the Sine of the Angle DEA : Wherefore the Sine of the Angle EDO is triple of the Sine of the Angle HDC , and consequently (in these very small Angles, which have the same *Ratio* with their Sines) $1\frac{1}{2}$ of the Sine of the Angle ADC . Therefore DC (or De lying in the same right Line with it)

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is the refracted Ray of ED passing out of Air into Glafs.

When the Ray passes out of Glafs into Air (as in the four last Figures) the *Focus* will be found from the same Principles, by taking the Point C such, that half the right Line EA may be to AB , as EC to BC .

C O R O L. I.

From this *Prop.* and XII. and *Corol.* I. *Prop.* XI. it will be easy, from the given *Focus* of Rays converging, and falling upon a spherical Surface of a *Medium* of different Density, to find their *Focus* after Refraction.

C O R O L. 2.

Hence, from the *Foci* E and C , and the *Vertex* B being given, we may find A the Center of the Sphere, and thence the refracting Sphere it self; by taking the Point A such, that EA may be to AC in a *Ratio* compounded of the *Ratio's* of BE to BC , and I to R : Or, from the *Foci* and Center being given may be found the *Vertex*. The like Problem concerning spherical *Specula*, may (by the *Correspondent Prop. in Catoptrics*) be solved with equal Ease, and more elegant Geo-

Geometrical Constructions from thence deduced.

C O R O L. 3.

The Geometrical Construction of this *Prob.* X. is easily deduced from what has been premised, *Plate II. Fig. 16.* Every thing else remaining as before, through *B* and *A* draw the right Lines *BQ*, *AN* parallel, meeting the right Line *EN* drawn from *E* as you please in *F* and *N*: Make *AN* to *AM*, as *I* to *R*. Join the right Line *FM*, and produce it till it meets the *Axis EA* in *C*. This will be the *Focus* required.

Join *CN*, meeting the right Line *BF* in *Q*; to this, through *F*, draw parallel the right Line *FH*, intersecting the right Line *EA* in *H*.

For $EA. EB :: (EN. EF :: EC. EH :: EC - EA. EH - EB ::) AC. BH.$ & permutando $EA. AC :: EB. BH.$ Therefore $CB. BE, + EA. AC :: (CB. EB, + EB. BH :: BC. BH :: BQ. BF :: AN. AM ::) I. R.$ Whence *C* is the *Focus* required.

This same Construction will serve (only changing the Order of the Points *E*, *B*, *A*, *C*, *M* and *N*) if *R* exceed *I*, or if the Rays converge, or are parallel; in which Case it will be changed into that of *Prob.* IX.

Or if the Concavity of the refracting Surface look towards the *Focus* E, or if B D be plane, in which Case it changes into that of *Prob.* VIII. For when the right Line A M N (divided as above) is infinitely distant from B Q, the right Line F M will be the same in Position with a right Line joining the Point F with a Point dividing B E, after the same manner as A M is divided in N, supposing B E to be homologous to A M; because the Tangents of Angles are reciprocally as the Tangents of their Complements to a right Angle.

After the same manner, if of the four Points A, B, C, and E, any other three be given, *Ex. gr.* E, B, and C, the fourth A will be determined; that is, the spherical Surface passing through the given Point, and changing the *Focus* E into C, will be determined; & *vice versa*.

SCHOLIUM.

Since there is no Spheroid or Conoid generated by the Rotation of a conic Section round its *Axis*, and consisting of a *Medium* of different Density from the ambient *Medium*, which can exactly change a given *Focus* of Rays into another given one by Refraction at a single Surface; it follows, that this Property of performing the Thing proposed pretty nearly, does therefore belong

long to a spherical Surface; because the greatest Circle of that Sphere has the same Degree of Curvity with *Cartes's Curve*, (*Lib. II. Geometriæ*) by whose Rotation are made Surfaces, which, separating *Media* of different Density, answer the Problem exactly. But if the Condition of that equicurve Circle, and of the given *Foci* lying on the same Side of the Center, be such, that the *Semidiameter* of the Circle be a Middle proportional between the Distances of the *Foci* from the Center of the Circle; and one of these Distances be to the *Semidiameter* in a *Ratio*, measuring the Refraction between the given *Media*, that Curve of *Cartes* is changed into the Circumference of a Circle. In this Situation of the *Focus* a Sphere of given Density, generated by the Rotation of a Circle, will so refract Rays proceeding from one of the *Foci*, that they shall all afterwards diverge precisely from the other of them.

PROP. XVI. PROB. XI.

ONE Focus of a given Lens being given, to find the other.

A *Lens* is a transparent Body, of a different Density from the ambient *Medium*,
and

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and terminated by two Surfaces, either spherical, or plane and spherical. A right Line perpendicular to both its Surfaces is called the *Axis* of the *Lens*. The Points where the *Axis* intersects each Surface are called the *Vertices*, either the *Vertex* of *Incidence*, or the *Vertex* of *Emersion*, according as it lies in that Surface upon which the Rays first fall, or out of which they again emerge. The *Thickness* is the Distance between the *Vertices*.

The Terms being thus explained, the *Focus* required may, by means of a Calculation (which in Practice is to be preferred to the nicest Constructions) grounded upon the foregoing Propositions, be easily determined, and Canons (which, because of their Difficulty to be remembered, are here omitted) established, by making the Calculation general. As if from the *Focus* before Incidence being given, it were required to find the *Focus* after Emersion: First, the *Focus* of the Rays, after their Refraction at that Surface of the *Lens* upon which they first fall, must be found; and this is done by *Prop. XIII.* if the Surface of the *Lens* be plane; but if it be spherical, and the Rays parallel, by *Prop. XIV.* and by *Prop. XV.* if they be diverging or converging. And having thus got the *Focus* of the Rays, after Refraction at this first Surface, that is, while they are
passing

passing through the *Lens*, which is likewise called the *Focus of Transition*, after the same manner their *Focus*, after Refraction at the second Surface of the *Lens*, or rather at the Surface of the ambient *Medium* contiguous to this second Surface, will be found; that is, their *Focus* after Emerision from the *Lens*. Q. E. I.

If there were more than one *Lens*, we must proceed after the same manner with every one of them.

By the like Method, from the *Focus* made by the Help of one or more given *Lenses* being given, the *Focus*, before Incidence is found, or from the optical Machine, being given, the Distance of the Object is determined.

COROL.

If the Geometrical Construction of this Problem be desired, it is easily deduced from *Corol.* 3. of the foregoing *Prop.* by assuming the Construction there given for one Surface of a *Lens*, and repeating it for the other, *Fig.* 17. For the right Lines E N, B F, A N, and F M, being drawn according to the Directions of that *Corol.* 'tis plain the right Line F M will tend towards the *Focus* of Transition of those Rays, whose *Focus* before Incidence was E. If therefore this meets with the parallel
right

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right Lines, drawn through a , the Center of the hindmost Surface, and b , the *Vertex* of Emerfion, in n and f , and $a n$ be taken to $a m$, as I to R at the Egrefs out of the *Lens*; (that is, as R to I at the Ingrefs into the *Lens*) the joined right Line $m f$ will meet the *Axis* $B b$ in e , the *Focus* required, after Refraction at both the Surfaces of the *Lens*: For the right Line $m f$ is in the fame Condition, with refpect to the Emerfion of the Rays from the *Lens*, in which M F was with refpect to their Immerfion.

In like manner, if of the fix Points, A, B, E, a , b , e , any other five be given, the fixth may be determined: For Example; From the two *Foci*, the Thicknefs of the *Lens*, and one Surface being given, the other Surface may be difcovered.

This is the Construction which *Barrow* received from a Friend, and placed, without any Demonftration, at the End of *Lect.* XIV.

PROP. XVII. THEOR. VI.

IF a plane radiant Surface fends out Rays upon any plane or fpherical re-
fracting Surface, the Rays proceeding from
every

every Point of the radiant Surface will, after Refraction, have their respective Foci very nearly in one and the same Plane, parallel to the radiant Plane. Plate II. Fig. 18.

Let any refracting Surface be expressed by $B D$, whose Center is A , and a plane radiant Surface by the right Line $E F$. From A upon $E F$ let fall the Perpendicular $A E$ meeting $B D$ in B . Find C , the *Focus* of those Rays after Refraction, whose *Focus* before Incidence was E ; through which draw the Plane $C T$ parallel to the Plane $F E$. I say, the *Foci* of Rays proceeding from every Point of the Plane $E F$ (or the principal Image of the Plane $E F$, which is made up of the Images of every one of those Points, with respect to an Eye placed in the *Axis* of Radiation: For we take no Notice here of the secondary Image seen by an Eye in any given Situation, which *Barrow* has considered in his three last *Lect.*) will all be posited very nearly in the Plane $C T$. From the Point F , taken at Pleasure in the radiant Plane, to the Center A , draw the right Line $F A$, meeting the refracting Surface at D , and the Plane $C T$ in T . $D C$ will be the refracted Ray of the Incident $E D$; and suppose the refracted Ray of the Incident $F B$ to be $B H$, meeting the right
L. Line

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Line DT in H . Because the Angle EAF is supposed but small, the Arch ED is to be looked upon as a right Line; and a Circle described on the Diameter BF will pass through the Points D and E , because of the Angles BEF , BDF being right. Whence the Angles EBF , DBF (namely the Angles of Incidence of the Rays FB and ED) contained in the same Segment, are equal; and consequently the Angles of Refraction, ABH , ADC , will also be equal. Therefore, by reason of the Angles at A equal, the Triangles BAH , DAC are equiangular; and BA is to AH , as DA to AC ; and since BA is equal to DA , AH will be likewise equal to AC : But because the Angle EAF , or TAC , is very small, AT is very nearly equal to AC ; and therefore AH , AT may be looked upon as equal; that is, the *Focus* of the radiant Point F is situated very nearly in the Plane CT . And since F is taken any how, the same will hold true of all the Points of the Plane EF ; namely, that their *Foci* will be in the Plane CT . Which is demonstrated after the same manner in any other Case whatever.

Q. E. D.

C O R O L.

Hence it follows, that the Image of the radiant Plane EF , to which the *Axis*
of

of the *Lens* is perpendicular, is also a Plane parallel to the former Plane : For the Image constituted in the Plane *CT* may be considered as a plane Surface sending out Rays upon the second Surface of the *Lens*. But if the Angle *EAF* be too great, so as that *AT* shall much exceed *AC*, and can by no means be looked upon as equal to it; in that Case 'tis easy, after the manner of *Corol. Prop. VIII.* to determine whether the Image of the Plane *EF* will be convex or concave : For Example ; It will be concave towards *A*, if *BD*, the refracting Surface, be of a denser *Medium*, and convex, or of a rarer *Medium*, and concave ; *& vice versa.*

PROP. XVIII. THEOR. VII.

THE Image appears from the Vertex of Emerſion under an Angle equal to that under which the Object appears from the Vertex of Incidence. Plate II. Fig. 19.

Let *GB*, the *Axis* of the *Lens*, be ſuppoſed produced, and ſtanding perpendicularly on the radiant Plane *FEH* at *E*, which (by *Corol.* of the foregoing *Prop.*) is alſo perpendicular to its Image *fCh*. Join

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the right Lines BF , BH , Gf , Gb . I say, the Angles $F B H$, $f G b$ are equal. Out of the innumerable Rays proceeding from the Point F , and after Refraction at the *Lens*, again united at f , the Image of this Point, choose two, one of which, FB , meets the *Lens* at the *Vertex* of Incidence B , and being there refracted, tends to ϕ , the *Focus* of Transition of that Point; and being again refracted at L , is directed towards f : The other, FD , being first refracted in D , tends straight on to ϕ , till it emerges out of the *Lens* at the *Vertex* of Emerision; where being again refracted, it proceeds towards f .

DGB is the Angle of Incidence of the Ray DG , and CGf its Angle of Refraction; and LBG is the Angle of Incidence of the Ray LB , which (by *Corol. 1. Theor. IV.*) would be refracted into BF , and EBF is its Angle of Refraction. Because the right Lines ϕB , ϕG (BG , the Thickness of the *Lens*, being neglected) become equal, the Sines of the Angles DGB , LBG , which are proportional to these, and consequently the Angles themselves, DGB , LBG , and therefore their Angles of Refraction, likewise CGf , EBF , will be equal. After the same manner CGb , EBH are found equal; therefore the Angles $F B H$, $f G b$ are equal: Which is shewn

shewn after the same manner in any other Case whatever. *Q. E. D.*

C O R O L.

Hence it follows, that a radiant Line is to its Image made by a *Lens*, as the Distance of that from the *Vertex* of Incidence, to the Distance of this from the *Vertex* of Emerfion, or (the Thickness of the *Lens* being neglected) as their Distances from the *Lens*. But if the Radiant be a Surface, their homologous Lines will still remain in the same Proportion; but the Radiant will be to its Image in a duplicate Proportion of those Distances. Whence it will be easy to determine the Proportion which the *last Image* (which is immediately seen by the Eye) of an Object, made by the Mediation of one or more *Lenses*, bears to the Object it self.

From this *Prop.* it follows likewise, that a radiant Line and its Image are cut in the same Proportion by the *Axis* of the *Lens* produced.

P R O P.

PROP. XIX. PROB. XII.

THE *Position of a Radiant in the Axis of the Lens being given, to determine its Image made by a given Lens, with respect to an Eye placed in the Axis of the Lens.* Plate II. Fig. 20.

Let $PCQD$ be the *Lens* proposed; in whose *Axis* CD , produced at Pleasure, suppose the Radiant to be placed, to the extreme Points of which the right Lines CE , CF do tend. Let b be the *Focus* of Rays proceeding from that Point of the Radiant which lies in the *Axis*, after Refraction at both the Surfaces of the *Lens*, found by *Prop.* XVI. Through which draw the Plane ebf , to which CD is perpendicular. Then through D the right Lines Df , De being drawn parallel to CF and CE , 'tis plain (from the two foregoing *Prop.*) that f is the Image of that Point to which the right Line CF is directed, and e is the Image of that to which CE is directed. From whence it is manifest, that the Image fbe will be seen by an Eye placed any where in CD produced beyond b , and receiving the *Rays* from the respective *Points* of the Image diverging.

But

But it is to be observed, that every Point of the Image f b e does not, like the primary Radiant, send forth Rays every way, and into all Parts; but the Rays of each Point constitute a Cone opposed at the Vertex to that Cone, which has the forementioned Point for its Vertex, and the Lens that refracts the Rays for its Base: Whence, from the Situation of the Eye, and the Diameter of its Pupil being given, it will presently be known whether the Eye will receive the Rays of any given Point; that is, whether it will see that Point. Which Consideration must always take place likewise in Vision of Images made by a Speculum.

For Example; If the Radiant be vastly distant, and the Surface $P C Q$ plane, $P D Q$ convex; and the *Lens* made of Glass with Air all around it, $D b$ will (by *Corol. 3. Prop. XIV.*) be equal to the Diameter of the Sphere $P D Q$: For the Rays being parallel to $B C$, pass unrefracted through the plane Surface $P C Q$, upon which they fall perpendicularly; and the Image in respect to the Radiant is inverted.

But if $P C Q$ be convex, and $P D Q$ plane, $C b$ will (by *Corol. 1. Prop. XIII.* and *Corol. 1. Prop. XIV.*) be equal to the Diameter of the Sphere $P C Q$, together with a third part of $C D$, the Thickness of the *Lens*. And neglecting the Thickness of
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the *Lens*, as is usually done in the object *Lens* of a Telescope, the Distance of the Image of a vastly distant Radiant, from a Plano-convex *Lens*, is equal to the Diameter of the Sphere.

If the Plano-convex *Lens* P Q were of Water, D b would, by *Prop. XIV.* be sesquialteral of the Diameter.

If the *Lens* P Q be of Glass, and both its Surfaces equally convex, the Thickness being neglected, D b will (by *Cerol. i. Prop. XIV.* and *Prop. XV.*) be equal to the *Semidiameter* of either Sphere. In an entire Sphere of Glass, the Image of a very distant Radiant will be at the Distance of a Quarter of the Diameter behind the Sphere; for in this Case the Thickness of the *Lens* cannot be neglected: In one of Water this Distance equals the *Semidiameter*.

In all these and the like Cases, if the Sun be that distant Radiant, and the *Lens* be notably broader than the Image of the Sun, in the Place of the Image a Burning will be excited; and that more vehement than from a concave *Speculum*, if the Image of the Sun be equally distant from a *Lens* and *Speculum* equally broad, because of the greater Loss of Rays at this than at that. If a lucid Body be put in the forementioned Place, the Image of it will be cast at a very great Distance, and
will

will enlighten those Parts that are vastly remote.

If the Radiant be not vastly distant as before, but yet more remote from the *Lens*, than the Place of the Image of a vastly distant Radiant; besides the Appearances just explained, namely, that the Image will be inverted, &c. if the Radiant approaches towards the *Lens*, the Image will recede, and *vice versa*; till the Radiant comes to the Place of the Image of a very distant Radiant, and then its Image will become vastly distant. All which may be seen in a darkened Chamber, receiving no Light but through a convex *Lens*, upon which Radiants at different Distances cast their Rays. The Place of the Image of any Radiant is known, by its being painted most distinctly upon a white unpolished Plane, situated in the Chamber. Nor is there need of subjoining any thing more of this Experiment, which is now very common; or of that other, founded upon the same optical Principles, in which, by the pellucid Colours of a Picture painted upon Glass, and transmitting the close Rays of a Flame, an Image is shewn painted upon a white unpolished Plane.

If the Radiant be nearer the convex *Lens* than the Image of a vastly distant Radiant, then its Image will be formed not on the

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opposite,

opposite, but on the same Side of the *Lens*, and its Place according to what has gone before, from the Place of the Radiant being given will be determined. This Image is always erect, and greater than the Radiant: And as the Radiant approaches to the *Lens*, the Image likewise approaches to it; and in like manner they both recede from it at the same time, but the Image more swiftly.

Fig. 21. If the convex *Lens* were changed into a concave one, the same Construction remains; and by a Calculation (according to the *Corollaries* of *Prop. XIV.*) $e b f$, the Image of a very distant Radiant, made by a Plano-concave *Lens* $P Q$, will be found erect, and on the same Side of D with the Radiant, and distant from it by the *Diameter* of the Sphere $P D Q$. Now if the Surface $P C Q$ be also spherical, and the Radiant from being distant becomes near, its Image will be determined by the same Construction, if by *Prop. XV.* the Image of the radiant Point placed in the *Axis* of the *Lens* be first found. In which Case, besides what has been just now said, it is also to be observed, that the Image approaches to, or recedes from, the *Lens* at the same time as the Radiant does, but more slowly; as will be plain to any one who will give himself the Trouble of a Calculation.

Let

Let the Rays of any Radiant, inflected after such a manner as to be ready to form the Image ebf if nothing hindered, be conceived to be intercepted by a Plano-concave *Lens* PQ ; upon whose concave Surface PDQ , described on a Diameter equal to the right Line Db , they first fall: The same Construction as before determines the Image made after the Refraction of these Rays at the forementioned *Lens*: For (by *Corol. 4. Prop. XV.*) the Rays converging towards b are so refracted from the concave Glass, as to become parallel to the *Axis*, that is, they constitute a vastly distant Image, to whose extreme Points (by the foregoing *Theor.*) the right Lines EC , FC , or eD , fD , are directed, and which is consequently given in Position. And in this Case the chief Thing to be observed is, that, to an Eye placed about B , the distant Image appears inverted, in respect to the Image that would have been made at ebf , without the Intervention of the *Lens*. It will be easy to proceed in all other Cases, according to these Examples. *Q. E. F.*

PROP. XX. PROB. XIII.

TO find such a Position of the Radiant with respect to a given Lens, that the Image made by the Lens may be equal to a given Figure, which is similar to the Radiant: Or, which is the same thing, that the Radiant may be to its Image made by the Lens in a given Proportion. Plate II. Fig. 22.

Let a *Lens* of Glass be given; for Example; Let AB be the *Semidiameter* of the first Surface, and CB the *Semidiameter* of the second Surface. Draw CM at Pleasure, making any Angle with CA . Let the Proportion of MD to DC be given; namely, that which the homologous Lines of the Radiant and its Image bear to one another. Join AM ; to which through B draw BE parallel, meeting the right Line CM in E ; ME taken twice will be the sought Distance of the Radiant from the *Lens*. If Dm be taken in the same right Line, but on the other Side of the Point D , equal to the right Line DM , and you join Am ; Be , drawn parallel to this, will cut off another me , which will likewise satisfy

satisfy the Problem: For the Radiant placed at the Distance of twice me from the *Lens*, is to its Image (but then it will be made on the contrary Side of the *Lens* with respect to the former Image) as DM to DC .

By *Corol. Prop XVIII.* the radiant Line is to its Image made by a *Lens*, as their Distances from the *Lens*: But if twice ME , or twice me , be the Distance of the radiant Point from the *Lens*, the Distance of the Image of that Point from the *Lens*, will be to the former Distance respectively, as MD to DC ; as will be plain to any one, who will undertake a Calculation according to *Prop. XV.* For if AB be called a , BC b , the Thickness of the *Lens* C , MD r , and z the sought Distance of the Radiant from the *Lens* of Glass about which Air is circumfused; we shall have $zz =$

$$\frac{6abz - 2acz + 6arz \times 2crz + 4acr}{3a - c + 3b}$$

If the Thickness of the *Lens* be neglected (which is done in the Construction of the Problem) the sought Distance of the Radiant from the *Lens* will be equal to

$$\frac{2ab \times 2ar}{a + b}$$

In the Scheme referred to the *Lens* is made convex on both Sides; but the same Construction will serve for any *Lens*, since from the Variety of *Lenses* only the Order of the Points A, B, and C is changed. Another Construction of this Problem is easily deduced from *Corol. Prop. XVI.*

PROP. XXI. PROB. XIV.

W*ith two given spherical Lenses or Specula, or one Lens and one Speculum, to make an optical Machine, which, to an Eye seeing at a given Distance, shall distinctly represent a given near Object under a given Angle, the Distance of the Eye from the last Lens or Speculum being likewise assigned.*

Since the Eye is a Machine made on purpose, that the Images of exterior Radiants may be distinctly painted upon its Bottom (which is made concave for this End, as *Corol. Theor. VI.* requires) all the rest of its *Apparatus* conducing only to its Motion or Security, which are necessary to be provided for; it is plain, that a given Eye can only see distinctly at a given Distance from the Object. Now if the Eye could place
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it self at Pleasure, at such a Distance from any Object as is necessary to distinct Vision, (all other things, as the Degree of Light, &c. being as they should be) Vision would always be distinct. And though there be no Eye so stiff, as to see only at a determinate Distance, but can, according to the Flexibility or Mobility of the Parts with which every Eye is endued, apply it self to Objects placed at different Distances, and change its Figure according to the Distance given; so as to be no longer looked upon as the same given Eye, but various and mutable, as Occasion requires: Yet, since this Mobility is confined within certain Limits, and there are a great many Objects to which we cannot at Pleasure come near enough to be within those Limits; 'tis plain, there will be need of an optical Machine to see them distinctly. But any spherical *Lens* or *Speculum* will be sufficient for this Purpose; since by its Assistance the Image of any Radiant (to which we cannot come so near as we would) may be brought near us (as is plain from what has been before demonstrated) and then we shall be able to view it, since the Eye is supposed moveable at Pleasure with respect to any thing near.

But because, besides distinct Vision, our Occasions sometimes require us to look into the more minute Parts of an Object, and
it

it is found by Experiment, that an Object seen under a less Angle than of one Minute, is considered by the Observer as a Point, and its Parts not at all to be distinguished one from another: It often happens, that when the Object is brought nearer the Eye, that the Particles to be observed may be seen under a sensible Angle, and greater than the forementioned one, the Object it self becomes too near the Eye, and is out of the Limits required for distinct Vision. This Inconveniency, if it be the only one, may be remedied by the Assistance of any given *Lens* or *Speculum*, by *Prop.* X. or XX. where the Image of an Object made by a given *Speculum* or *Lens* is represented in any given Measure.

But if both the forementioned Inconveniencies urge at the same time, they are not to be removed without the Help of two *Lenses* or *Specula*, or one *Lens* and *Speculum*. Having shewn therefore what Assistance the Sight may receive from a single given *Lens* or *Speculum*, we shall proceed to Machines made by two combined together; or to construct the Problem proposed universally. *Plate II. Fig. 23, 24.*

Let R be the given Object, S the given Angle in which it is to be represented, D the given Distance requisite to distinct Vision, and L the given Distance of the Eye from

from the *Lens*. Make the Triangle $A O B$, in which the Angle at the *Vertex* O is equal to the given S , and $O E$ perpendicular from the *Vertex* upon the *Basis* equal to the given D . If we take the middle Point of the Object placed in the *Axis* of the *Lens* or *Speculum*, which in Practice is very convenient, the Triangle $A O B$ must be made Ifofceles. Take $O V$ equal to L . At V place either of the *Lenses* or *Specula*, having its *Axis* in the right Line $O E$.

By *Prop.* VI. or XVI. having one *Focus* E of a *Lens* or *Speculum* given, find the other e ; that is, that the Rays whose *Focus* before Incidence is e , may have E for their *Focus* after Inflexion at the *Lens* or *Speculum*. Through e draw the right Line $a e b$ parallel to the right Line $A E B$, meeting the right Lines $V a$, $V b$, drawn through V parallel to the right Lines $V A$, $V B$, in a and b .

'Tis plain, from *Prop.* IX. or XIX. that if $a e b$ be the Radiant, $A E B$ will be its Image: Wherefore if the given near Object R , and another *Lens* or *Speculum*, be placed after such a manner (by *Prop.* X. or XX.) that the Image of the Object R made by this *Lens* or *Speculum*, may obtain the Situation and Magnitude $a e b$; the Microscope required is made: For $a e b$ is the Image of the Object R , the

Image of which Image, seen by the Eye placed in O, is A E B. Now this appears under the Angle A O B equal to the given S, and at the Distance O E equal to the given D, and consequently distinct ; and the Distance of the Eye from the last *Lens* or *Speculum* in V, is equal to the given L. Q. E. F.

If the given Eye be an old Man's, every thing else remaining as before, the right Line O E becomes infinite, and the Point *e* is found by *Prop.* III. or XIV.

If the Object proposed were vastly distant, the Problem would be impossible. But if the Angle S were not given, a Telescope from the remaining *Data* might thus be made. In the right Line given in Position, tending towards the proposed distant Object, straight forward from O, take O E, O V, equal to D and L : And in V place one of the *Lenses* or *Specula*, having its *Axis* in V E. By *Prop.* VI. or XVI. one *Focus* E of the *Lens* V being given, find the other *e* ; which being supposed the *Focus* of Rays before Inflexion at V, E may be their *Focus* after Inflexion. In the right Line O E let the other *Lens* or *Speculum*, having its *Axis* in the same, be so placed, that the Image of the distant Object made by it may be situated in the right Line *a e b* perpendicular to O E, and the Telescope required is made. For the first

first Image of the distant Object is in aeb : And the Image of this Image is, by Construction, in AEB , whose Distance from O is equal to the right Line OE or the given D , and consequently distinctly seen; and the Distance of the Eye from V is equal to the given L . But a distant Object being given, by *Prop. XIX.* its Image made by a given *Lens* or *Speculum*, and likewise the Image of that Image made by the given V will be given, and consequently the Angle under which this last is seen by the Eye in a given Position.

But if O be an old Man's Eye, the Angle AOB , because of AB being in this Case vastly distant, is equal to AVB , or aVb . Therefore the Angles under which a distant Object with and without a Machine appears, are as the Distances of the *Lenses* or *Specula* from the common *Focus e*: For small Angles are almost as their Cotangles.

SCHOLIUM.

It will be very convenient that the *Lens* or *Speculum*, which immediately receives the Rays of the Object, and forms its first Image (and is therefore called the *Object Glass*) be as perfect as possible: For the Errors or Defects of this *Lens* or *Speculum*

affect the Image made by it. And since this Image acts the Part of an Object, to be seen through the *Speculum* or *Lens* V, (which is nearest the Eye, and therefore called the *Ocular Glass*) its Defects, that is, the Defects or Errors of the Object Glass by which it is formed will be greater and more sensible, by how much the Image AEB is greater than *aeb*; that is, by how much more perfect (the Object Glass remaining the same) the Machine is made. But the Errors of the Ocular *Lens* or *Speculum* V, are equally sensible, whatsoever be the Image *aeb*, or the Object *Lens* or *Speculum* by which it is produced; that is, the Ocular Glass V remaining the same, its Defects are equally apparent and discoverable, to whatsoever Degree of Perfection the Machine, by changing the Object *Lens* or *Speculum*, which forms the Image *aeb*, be brought: For the Eye being given, the given Ocular *Lens* or *Speculum* V is always at the same Distance from the Image *aeb*, doing the Office of an Object and its Image AEB, and consequently shews the same Defects.

PROP. XXII. PROB. XV.

With three or more given spherical Lenses or Specula, to make a Machine, which to a given Eye shall distinctly represent a proposed distant Object under a given Angle, the Distance of the Eye from the last Lens or Speculum being assigned.

By the Help of any one of the given Lenses or Specula from the Image of the distant Object, and with the two others, by the foregoing Prop. make a Microscope that shall represent it in the Conditions proposed, and the Telescope required is made.

In like manner with these Lenses or Specula may a Microscope be made; and then by adding a fourth, another Telescope: In all which we have the Proportion which the Image seen by the Eye bears to the Object, or which the Angle under which that is seen, bears to the Angle under which this is seen without the Machine: And consequently the Powers of a Machine in promoting Vision, are, by Corol. Prop. IX. and XVIII. easily estimated.

PROP. XXIII. PROB. XVI.

TO make a spherical Lens of Glass, whose Thickness is given, that shall to an Eye seeing at a given Distance represent a given Object placed at a given Distance, under a given Angle distinctly, the Distance of the Eye from the Lens being likewise assigned. Plate III. Fig. 1.

We have hitherto shewn what Assistance may be had from given *Lenses* or *Specula*, or both, howsoever combined, in order to supply the Defects of Vision; it remains, that we demonstrate the Manner of making a *Lens* for given Uses.

The radiant Line AB , whose Distance from the *Lens* is VE , is to be represented distinctly under the visual Angle aOb , to an Eye seeing distinctly at the Distance Oe . In the Isosceles Triangle aOb (whose Base is ab , and Height eO) upon the Perpendicular Oe (so situated as to bisect the radiant Line AB perpendicularly in E) take OL equal to the given Distance of the Eye from the *Lens*. Draw aL , bL , and produce them to D and F . Make LV equal to the given Thickness of the *Lens*. From the right Line VL , given
in

In Position, perpendicular to the Glafs at the Points of Incidence, draw the refracted Rays in Glafs $V\alpha$, $V\beta$, LH , LG corresponding to those in Air AV , BV , DL , FL . Produce the right Lines HL , GL , till they meet with the right Lines $V\beta$, $V\alpha$ in β and α . The right Line $\alpha\beta$ join'd, will cut the Perpendicular eO perpendicularly in ϵ , because both the Angles of Incidence and of Refraction, on each side of the right Line Oe are equal. From the *Foci* E and ϵ , and the Vertex V being given, find (by *Corol. 2. Prop. XV.*) the Sphere of Glafs KVM that may refract the Rays in Air diverging from E , so as to make them converge towards ϵ . By the forementioned *Corol.* find also NLP , the Surface of a Sphere of Air passing through L , that may refract the Rays in Glafs converging towards their *Focus* ϵ , so as to make them afterwards diverge from e : The solid Figure $MKNP$, being made of Glafs, and terminated by the spherical Surfaces KVM , NLP , and a conic Surface whose *Axis* is VL , is the *Lens* required. But Care must be taken, that the Portions VK , LN be not too great; because all the preceding Demonstrations hold only true of Rays falling near the *Vertex*.

By Construction, the Rays diverging from E do, after Refraction at the first Surface

Surface KVM , converge to the *Focus* ϵ : Wherefore (by *Prop.* XVII.) the Image of the radiant right Line AB (made by Refraction at KVM) is in the right Line $\alpha\beta$ given in Position: But the Ray AV , by Construction, after it is refracted at the Surface KVM comes to the Point α ; therefore that Point α is the Image of the Point A , which is produced by the first Surface only. In like manner β is the Image of the Point B ; and the right Line $\alpha\epsilon\beta$ is the Image of the radiant right Line AEB . Moreover (by Construction) the Rays within the *Lens* that converge towards ϵ , after they are refracted at the Surface NLP , diverge from e ; wherefore (by *Prop.* XVII.) the Image of the future Image $\alpha\epsilon\beta$, made by Refraction at the Surface NLP , is in the right Line aeb : But the Ray GL within the *Lens*, proceeding directly towards α , after it is refracted, becomes LF , and proceeds from a ; whence the Image of the future Image in α , after Refraction at NLP , becomes a : And so likewise the Rays within the *Lens* that converge towards β , after Refraction at NLP , diverge from b ; that is, aeb is the Image of the Radiant AEB , placed at a given Distance VE from the *Lens*, made by the *Lens* $NKMP$, and to be seen distinctly by a given Eye in O ; because it is

at the required Distance Oe from the Eye, and is also seen under the given Angle aOb by the Eye O , which is at the given Distance OL from the *Lens*, and VL is the given Thickness of the *Lens*. *Q. E. F.*

The same Construction serves, if the Eye of an old Man be given: For in that Case aeb is at an infinite Distance, and the former right Lines aL , bL must be drawn through L parallel to the right Lines Oa , Ob given in Position, and the Center of the Surface NLP (by *Corol. 6. Prop. XIV.*) is distant from L by a third part of the right Line Le .

But if the Object proposed be a distant one, the Construction will become much more simple: For the Center of the first Surface KVM (by *Corol. 1. Prop. XIV.*) will be distant from the *Vertex* V by a third part of the right Line Ve .

PROP. XXIV. PROB. XVII.

TO make a concave spherical Speculum of Glass, whose Thickness and Diameter of Concavity are given, so that the Rays parallel to its Axis, reflected from both its Surfaces, shall be collected
O
in

in the same Point of its Axis. Plate III.
Fig. 2.

Because it is found by Experience, that *Specula* made of Metals neither receive a due Figure and Polishing so easily, nor preserve it so long, it will be convenient to use those of Glass. Let B then be the *Vertex* of the concave Glass Surface EBF, A its Center, and BD the Thickness of the *Lens*, be given; we are to find DC the *Semidiameter* of the Surface GDH, so that the Rays parallel to AB, and falling upon the concave *Speculum*, as well those that are reflected from the Surface EBF, as those that are twice refracted at the forementioned Surface EBF at their Entrance into it and Emerision out of it, but reflected from the concave Surface GDH, may all meet in the same Point of the *Axis* AB: Or, which is the same thing, a *Speculum* of Glass is to be made after such a manner, that the two Images of a distant Object, made by the two Surfaces of the *Speculum*, may coincide, and consequently so as to be most powerful in burning, or in forming the Images of distant Objects. *Plate III. Fig. 3.*

From any Point L of the infinite right Line SM, on one Side take LN equal to
BD,

B D, and on the other Side L R equal to twice B D; from N likewise directly forward take N T equal to thrice A B, and T V equal to twice N T, and V M also equal to twice R L. On the Center R, with the *Radius* R T, describe a Circle meeting the Perpendicular erected at L in K. Join M K; and at K erect the Perpendicular K S meeting the infinite right Line first drawn, in S, *Fig. 2.* In the right Line D A, from D towards A, take D C equal to the right Line L S; and on the Center C with the *Radius* C D describe an Arch of a Circle D G, similar to the Arch B E described on the Center A, and join E G. The solid Figure, generated by the Rotation of the plane Figure B D G E about the fixed right Line B D, and consisting of Glass, is the *Speculum* required. *Q. E. F.*

Fig. 4. But if A B be sufficiently large in respect to the Thickness B D, the Problem will admit of this more expeditious Construction. Divide B D in O, so that the greatest Segment B O may be to the lesser O D, as 5 to 4. Produce B A to C, so that C A may be equal to O D; and C will be very nearly the Center of the outward Surface G D H. *Q. E. I.*

We shall omit the Demonstration of these two Constructions ; because it is very easily deduced from the following analytical Calculation founded upon *Prop. V. XIV.* and *XV.* For if $AB = a$, and $BD = c$, we shall have $DC = \frac{9aa + 18ac + 5cc}{9a + 5c}$.

In like manner may a *Speculum* be made, if the Radiant be in any other Position.

I had determined to have subjoined a general Calculation for finding the *Foci* of any *Speculum* or *Lens* universally : But that is abundantly done already for *Lenses* by that excellent Analyst EDM. HALLEY, in the *Philosophical Transactions* for November, 1693, and elegantly applied to particular Cases.

SCHOLIUM.

Hitherto we have shewn what Advantages may be expected from spherical *Lenses* or *Specula*, towards the Construction of Machines : But the different Refrangibility of the Rays of Light, and that in given Rays given, immutable, and annexed to certain Colours, discovered by that admirable Philosopher Sir

ISAAC

ISAAC NEWTON, has so much disturbed our Dioptrical Reasonings, that no Exactness can now be hoped for from *Lenses*, even though formed into what conoidical Figures we please. But since the Law of *Catoptrics* concerning the Equality of the Angles of Incidence and Reflexion, is preserved inviolable in Rays, however heterogeneous, as the same Great Man observes; it is better to use a *Speculum* instead of the Object *Lens*, which forming the Image of a distant Object at a considerable Distance, discovers the Errors that arise from the different Refrangibility of Rays sensible enough, and not at all to be dissembled,* if the Rays falling obliquely be admitted by an Aperture sufficiently large, which is very often necessary: But in smaller *Lenses*, such as the ocular ones, the Error is so small and insensible, that they may be still safely used.

James Gregory † was the first who gave a Specimen of this Sort of *Cata-Dioptrical Telescopes*, consisting of *Lenses* and *Specula*, in *Optic. Promot. Prop. 59.* which was many Years afterwards given out by Mr. *Cassegrain*, a *Frenchman*, for his own. The same, upon Physical as well as Geometrical Accounts, altered and improved,

* See the *Latin*.

† Our Author's Uncle.

110 *The Elements of Dioptrics.*

is published by Sir ISAAC NEWTON, in his admirable *Theory of Lights and Colours*.

Since *Specula*, being opaque Bodies, cannot have the same *Axis* with *Lenses* without being perforated at their *Vertex*, and consequently suffering an irreparable Loss (arising from both these Causes) of those Rays that fall near the *Vertex*, and are most accurately reflected: The Position of the *Axis* may (by *Corol. 1. Prop. II.*) be altered as we please, by the Help of a plane *Speculum*; and by this means (besides other Advantages) the Necessity of Perforation is quite taken away, and the Loss of Rays falling near the *Axis*, occasioned by the Opacity, is very much diminished by the Obliquity of the second *Speculum*, which is observed by the accurate *Newton* in his *Cata-Dioptrical Telescope*. But if, by reason of Physical Difficulties, in turning and polishing proper *Specula*, we must still continue the Use of *Lenses*, perhaps it would be of Service to make the Object *Lens* of a different *Medium*, as we see done in the Fabrick of the Eye; where the crystalline Humour (whose Power of refracting the Rays of Light differs very little from that of Glass) is by Nature, who never does any thing in vain, joined with the aqueous

aqueous and vitreous Humours (not differing from Water as to their Power of Refraction) in order that the Image may be painted as distinct as possible upon the Bottom of the Eye. There are likewise other Advantages of the forementioned Artifice in the animal Eye, which belong not to this Place.





A

SUPPLEMENT

To the foregoing WORK.



IN order to remove whatever Difficulty might lie in the Way of a Reader who is to begin with this Subject, and interrupt his Progress thro' this most excellent Treatise (besides having all along in the Translation explained such Passages as the Author's Laconic Style has made something too intricate for a Beginner, and corrected several considerable Faults of the Press, which had escaped his Care) I have thought it proper to subjoin a few things, which may perhaps be of Service to obviate some farther Difficulties, and supply some seeming Defects.

I. From

I. From Prop. VI. Prob. V.

We may give a Solution of the following Catoptrical Problem, for finding the Focus of any given Speculum universally; which the Author tells us, at the End of his Book, it was once his Design to have done.

The Problem is this. *The Focus, or Point in the Axis of a given Speculum, from whence, or towards which, Rays proceed, being given, to find the Focus or Point where those Rays will meet again, after they fall upon, and are reflected by, a given Speculum.*

Now this Problem being solved in one Case, namely, that of Rays falling upon a *convex Speculum*, and diverging from a certain Point in the *Axis* of the *Speculum*, will, *mutatis mutandis*, be applicable to all other Cases whatever, whether the *Speculum* be *convex*, *concave*, or *plane*, and whether the Rays fall *diverging*, *converging*, or *parallel*.

[*Plate I. Fig. 9.*] Let then *BD* be a given *convex Speculum*, whose Center is *A*, and *E* a given Point in its *Axis*, from whence the Rays which fall upon the *Speculum* diverge: It is required to find the *Focus*, or Point *C*, in which the Rays *ED* diverging from the *Focus*, or Point *E*, do, after Reflexion from the *convex Speculum BD*, meet. Call *EB* the given Distance of the Point *E* from *B* the *Vertex* of the *Speculum*

lum d , AB the given *Radius* of the *Speculum* r , and BC the Distance of C the *Focus* required from B the *Vertex* of the *Speculum*, x . By *Prop. VI. Prob. V.* $AC (r - x) : CB (x) :: AE (d + r) : EB (d)$. From whence arises the Equation, $dr - dx = dx$

$$\frac{dr}{dr} + rx,$$
 which gives us $BC (x) = \frac{dr}{2d + r}$

for our general Rule; which shews, that in the Case of *diverging Rays* falling upon a *convex Speculum*, the *Focus C* is always affirmative, and to be taken from the *Vertex B* directly forward: And that the greater d is, the greater will be the *focal Distance*, till at last d becoming infinite, and consequently the finite Term $+r$ vanishing,

it will be $x = \frac{dr}{2d} = \frac{r}{2}$ or half the *Radius*;

that is, the *Radiant* receding from a *convex Speculum*, the *Image* will also recede beyond the *Vertex* of the *Speculum*; but so slowly, that when the *Radiant* becomes vastly distant, the *Image* will be got no farther than the middle Point, between the *Vertex* and Center of the *Speculum*: And *vice versa*, the nearer the *Radiant* approaches to the *Speculum*, the nearer the *Image* approaches to it, till at last they both meet and coincide at the *Vertex*. According to *Observat. 2. Schol. Prop. VIII.*

If

If $EB (d)$ be equal to $AB (r)$, the

focal Distance BC becomes $= \frac{r}{3}$: But if

$EB (d)$ be equal to half $AB (\frac{1}{2}r)$, the
focal Distance will in this Case become
 r .

$= \frac{r}{4}$

Now to apply this universal Canon to all other possible Cases; the Terms of which it consists remaining always the same, 'tis only changing the Signs $+$ or $-$ according as the Case requires: For in the Case of *converging Rays*, the Point in the *Axis* of the *Speculum* towards which they converge, is on the other Side of the *Vertex* of the *Speculum*, and consequently its given Distance, in respect to the former, is *negative*, or $-d$. For the same Reason, in the Case of a *concave Speculum*, the Center of the spherical Surface lying on the contrary Side of the *Vertex*, the given *Radius* becomes *negative*, or $-r$. And in these Cases respectively, wheresoever the Quantity d or r occurs, it must have a contrary Sign to what it had before.

If then *converging Rays* fall upon a *convex Speculum*, d being in this Case nega-

tive, the Rule will be $\frac{-dr}{-2d + r}$; which

shews, that when $2d$ exceeds r , the *Focus* will be still affirmative; but if $2d$ be less than r , the *Focus* will be negative, or on the contrary Side of the *Vertex* of the *Speculum*: That is, if the Point behind the *Speculum*, towards which the Rays converge, be at a greater Distance from the *Vertex* of the *Speculum* than half the *Radius*, the *Focus* is still to be taken from B the *Vertex* directly forward, according to *Corol. Prop. IV.* But if its Distance be less than half the *Radius*, then the *Focus* must be taken from the *Vertex* B backward, according to *Corol. Prop. VI.* When d is equal to $-\frac{1}{2}r$, the focal Distance becomes $= \frac{-dr}{0}$

and is consequently infinite: That is, Rays converging to a Point in the *Axis* of a convex *Speculum*, at an equal Distance between the Center and *Vertex*, will, after Reflexion, proceed parallel, according to *Corol. Prop. III.* When d is equal to r , the focal Distance becomes $= r$; that is, Rays converging towards the Center of the *Speculum*, are reflected by a convex *Speculum* back again upon themselves.

If *parallel Rays* fall upon a convex *Speculum*, d in this Case becoming infinite, the Rule will be $\frac{dr}{2d} = \frac{1}{2}r$; which shews, that

Rays

Rays falling parallel upon a convex *Speculum*, are collected in a Point at the Distance of half the *Radius* behind the *Speculum*, according to what has been demonstrated at *Prop. III.* And consequently a vastly distant Radiant will have its Image formed in this Point; whence the Sun's Beams will be there collected, and heat or burn any thing placed therein.

If *diverging Rays* fall upon a *concave Speculum*, the *Radius* being in this Case

$$-dr$$

$-r$, the Rule will be $\frac{-dr}{2d-r}$; which shews

that when d is less than $\frac{1}{2}r$, the *Focus* is *affirmative*; when d is equal to $\frac{1}{2}r$, the *Focus* is *infinite*; and when d is greater than $\frac{1}{2}r$, the *Focus* is *negative*; and when d is equal to r , the *focal* Distance is $-r$: That is, if the Point in the *Axis* of the *Speculum* from which the Rays diverge be nearer the *Vertex* than half the *Radius*, the *Focus* will still be behind the *Speculum*, according to *Prop. V.* If it be just at the Distance of half the *Radius*, the Rays after Reflexion will proceed parallel, according to *Prop. III.* If it be at a greater Distance than half the *Radius*, the *Focus* after Reflexion will be on the same Side of the *Speculum* with the *Focus* before Reflexion, or Point from which the Rays diverge. If it be at the Distance of the whole *Radius*,
the

the Rays after Reflexion meet in the same Point from which they first diverge. It is moreover manifest, that the more d exceeds $\frac{1}{2} r$, the less will be the *negative focal Distance*; but if d be infinite, the

$$\text{focal Distance in this Case} = \frac{-dr}{2d} = \frac{1}{2} r,$$

can be no less than half the *Radius*: And on the contrary, the less d exceeds $\frac{1}{2} r$, the greater will be the *focal Distance*; till at last, d becoming equal to $\frac{1}{2} r$, the *focal Di-*

$$\text{stance in this Case} = \frac{-dr}{0} \text{ becomes infi-}$$

nite. But when d grows any thing less than $\frac{1}{2} r$, the *Focus* becoming *affirmative*, is thrown at a greater Distance on the contrary Side of the *Speculum*; and by how much d is less than $\frac{1}{2} r$, the less will be this affirmative *focal Distance*. So that a *Radiant* placed at a greater Distance than half the *Radius* from a *concave Speculum*, the farther it recedes from the *Speculum*, the nearer its *Image* which is on the same Side approaches to the *Speculum*, and at the Center of the *Speculum* they meet, and afterwards cross one another, till the *Radiant* becoming *vastly distant*, the *Image* will come within half the Distance of the *Radius* from the *Speculum*: And *vice versa*, the nearer the *Radiant* approaches the *Speculum*, the farther the *Image* recedes from it,

it, and at the Center of the *Speculum* both meet, and afterwards cross one another, till at last the Radiant coming to half the Distance of the *Radius* from the *Speculum*, the Image becomes *vastly distant*: Whence if a lucid Body be placed at the Distance of half the *Radius* from a concave *Speculum*, it will enlighten Places that are vastly distant. If the Radiant comes nearer the *Speculum* than half the *Radius*, the Image will be cast from before the *Speculum* to a great Distance on the contrary Side; and the nearer the Radiant now approaches to the *Speculum*, the nearer will the Image likewise approach to it, till at last they both coincide at its *Vertex*, and *vice versa*, according to *Observat. 1. Schol. Prop. VIII.*

If *converging Rays* fall upon a *concave Speculum*, d and r being in this Case both

negative, the Rule will be
$$\frac{dr}{-2d - r};$$

which shews that the *Focus* is always *negative*: That is, Rays that fall converging upon a concave *Speculum*, will always be collected in a Point or *Focus* on this Side the *Speculum*, according to *Corol. Prop. V.* If d be equal to r , the *focal* Distance is

$= \frac{-r}{3}$; but if d be equal to $\frac{1}{2}r$, the *focal*

Distance is $= \frac{-r}{4}$.

If

If *parallel Rays* fall upon a *concave Speculum*, d in this Case becoming infinite,

the Rule will be $\frac{-dr}{2d} = -\frac{1}{2}r$: That is,

Rays falling parallel upon a *concave Speculum*, are collected in a Point at the Distance of half the *Radius* on this Side the *Speculum*, as has been demonstrated at *Prop. III.* Whence if the Sun's Beams be received upon such a *Speculum*, the same will be the burning Point.

If *diverging Rays* fall upon a *plane Speculum*, the *Radius* r being infinite, the Rule will be $\frac{dr}{r} = d$; that is, the *Focus* of diver-

ging Rays reflected by a *plane Speculum*, will be at as great a Distance behind the *Speculum*, as the Point from which they diverge is before the *Speculum*, according to *Prop. II.* And consequently the Image of any Radiant, made by Reflexion from a *plane Speculum*, will be seen as far behind the *Speculum* as the Radiant is before the *Speculum* ; and they will both not only recede from and approach to the *Speculum* at the same time, but likewise keep an equal Pace one with another.

If *converging Rays* fall upon a *plane Speculum*, d being negative, and r infinite,

the Rule will be $\frac{-dr}{r} = -d$; that is, Rays con-

converging to a Point at a certain Distance behind a plane *Speculum*, will have their *Focus* at an equal Distance from the *Speculum*; according to *Corol. 1. Prop. II.*

If *parallel Rays* fall upon a *plane Speculum*, both *d* and *r* being in this Case in-

finite, the *focal* Distance $\frac{dr}{2d+r}$ will be

also infinite: That is, Rays falling parallel upon a plane *Speculum*, will be reflected back parallel.

It is worth observing, that the Consideration of *diverging Rays* relates to Objects that are near us, and such as we examine with our naked Eye, or by the Help of a Microscope: *Parallel Rays* are considered when we have to do with Objects vastly distant, and such as we look at through Telescopes. And *converging Rays* fall under our Consideration, when the Rays proceeding diverging or parallel from any Object are by one *Speculum* or *Lens* made to converge, and then intercepted by the Interposition of another *Speculum* or *Lens*, before they arrive at their Point of Convergence; which is of great Use in examining the Effects of optical Machines, made by a Combination of more than one *Lens* or *Speculum*, and constructing such as are proper for any assigned Purpose, where this Contrivance is often absolutely necessary.

There are two farther *Uses* of this Method; the first is, to determine what Degree of *Convexity* or *Concavity* is necessary for a *Speculum* to represent a given Object at a given *Focus*. And this is very easily done from the Equation first found $dr - dx = dx + rx$: For d and x being given, the *Radius* of

$$\text{Convexity } r = \frac{2dx}{d-x}.$$

Where it is plain,

that if x be greater than d , r will be a negative Quantity, and the Problem impossible for a convex *Speculum*; That is, if it be required to represent a given Object, at such a given *Focus*, whose Distance on the other Side of the *Speculum* shall be greater than the Distance of the Object on this Side; instead of a convex Glass, we must use a concave, whose negative *Radius* will be

$$= -\frac{2dx}{x-d}.$$

After the same manner the De-

gree of *Concavity* is found from the same Equation, only changing the Sign of r from $+$ to $-$: For d and x being given, we

$$\text{shall have the } \textit{Radius} \text{ of } \textit{Concavity } r = -\frac{2dx}{x-d}.$$

Where it is manifest, that if d be greater than x , r will be a negative Quantity, and the Problem impossible for a concave *Speculum*: That is, if it be required to represent

sent a given Object, at such a given *Focus*, whose Distance on the other Side of the *Speculum* shall be less than the Distance of the Object on this Side, instead of a concave *Speculum*, we must make use of a convex,

whose affirmative *Radius* is $= \frac{2dx}{d-x}$. And

in both Cases if d be equal to x ; then the *Radius* either of Convexity or Concavity

$$= \frac{2dx}{+d-x} = \frac{2dx}{0} \text{ will be infinite, and the}$$

Problem will be impossible for either a convex or concave *Speculum*: That is, if it be required that a given Object shall be represented at such a given *Focus*, whose Distance on the other Side of the *Speculum* shall be equal to the Distance of the Object on this Side, instead of a *convex* or *concave Speculum*, a *plane Speculum* is the only one that can be used. If d be infinite, the

Radius of Convexity becomes $\frac{2dx}{d} = 2x$;

but the *Radius* of Concavity $\frac{2dx}{-d} = -2x$,

will have a negative Value: Which shews, that the Problem is impossible in the Case of a concave *Speculum*; that is, a vastly

distant Object cannot be represented at any given *Focus* behind a concave Speculum; but may by a Convex, whose Radius of Convexity must be equal to twice the given *focal* Distance behind the Speculum. If x be infinite, then the Radius of Convexity

$$= \frac{2dx}{-\infty} = -2d \text{ has a negative Value, and}$$

the Problem is impossible for a convex Speculum; but the Radius of Concavity is

$$= \frac{2dx}{x} = 2d. \text{ Whence if we would have a}$$

given Object represented at an infinite Distance behind the Speculum, we can make use of none but a concave Speculum, whose Radius of Concavity must be equal to twice the given Distance of the Object.

Hitherto we have considered the *Focus* as affirmative, that is, behind or beyond the Speculum; but the same Rule, only changing

$$\text{the Sign of } x \text{ in the Equation } x = \frac{dr}{2d+r},$$

will equally hold if we would have the *Focus* negative, or on the same Side of the Speculum with the Object: For in the Case of a convex Speculum we shall have the Ra-

$$\text{dius of Convexity} = \frac{-2dx}{d+x} \text{ always of a negative}$$

gative Value, and consequently the Problem is always impossible for a convex Speculum; but in the Case of a concave Speculum, the

$$\text{Radius of Concavity will be} = \frac{-2dx}{-d-x}:$$

Which shews, that the Problem is always possible for a concave Speculum, be the Quantities d and x as they will. If we would have $d = x$, then the Radius of Concavity will be also equal to d or x : That is, a concave Speculum, whose Radius is equal to the Distance of the Object from the Speculum, will reflect the Image into the same Place with the Object.

The other *Use* of this Method is, from the *Image* and *Speculum* given, to find the *Distance* of the Object from the *Speculum*: That is, in the Equation first found $dr - dx = dx + rx$, x and r being given, we are to find d , which will consequently

$$\text{be} = \frac{rx}{r - 2x} \text{ in convex Specula, and in}$$

$$\text{concave} = \frac{rx}{r + 2x}. \text{ Whence 'tis plain, that}$$

in *convex Specula* the Problem will be impossible when x exceeds $\frac{1}{2}r$, but in *concave Specula* it will always be possible: That is, if the Image is to be at a greater Distance than $\frac{1}{2}$ the Radius behind the Speculum, it cannot

cannot be made by an Object placed before a convex Speculum at what Distance soever: But let the Distance of the Image behind the Speculum be what it will, it may be formed by an Object exposed at some certain Distance before a concave Speculum. If the given *Focus* be negative, or the Image on the same Side of the Speculum with the Object, then changing the Sign of x , in the forementioned Equation, we shall have, in the Case

of *convex Specula*, $d = \frac{-rx}{r+2x}$; and in the

Case of *concave Specula*, $d = \frac{-rx}{r-2x}$. So

that the Problem will always be impossible for *convex Specula*, and only possible for *concave* ones, when $\frac{1}{2}r$ does not exceed x , or when the Image is not nearer the Speculum than by half the Radius.

II. From Corol. Prop. IX.

WE may deduce a Solution of the following Catoptrical Problem, of magnifying or diminishing a given Object by a given Speculum in any assigned Proportion.

The Problem is this: To find at what Distance from a given Speculum it is necessary

cessary to place an Object, in order that the homologous Lines of the Image made by the Speculum may bear any assigned Proportion to those of the Object.

Since it is evident from this *Corol.* that the homologous Lines of the Radiant and Image are to one another as their Distances from the Speculum respectively: It follows, that if b to c express the Proportion which the homologous Lines of the Object and Image are to bear to one another, b will be to c , as d , the Distance of the Object, to x , the *focal* Distance of the Image. Whence, if we compare this Value of x with that delivered in the foregoing Problem, we

shall have $\frac{cd}{b} = \frac{dr}{2d+r}$: And consequently

the Distance required $d = \frac{br - cr}{2c}$ will be

our general Rule; and will, *mutatis mutandis*, extend it self to all possible Cases whatever, though in its present Form it regards the Case of *convex Specula* in particular: For *concave Specula* 'twill stand

thus $d = \frac{cr - br}{2c}$. If the Image be desired

on the same Side of the Speculum with the Object, x being negative, in the Case of

convex

convex Specula, 'twill be $d = \frac{-br - cr}{2c}$;

and in the Case of concave $d = \frac{br + cr}{2c}$

From whence 'tis plain, that there is no magnifying an Object by a concave Speculum; for c being in this Case greater than b , the Rule for the affirmative Focus

$d = \frac{br - cr}{2c}$ will have a negative Value,

and that for the negative Focus

$d = \frac{-br - cr}{2c}$ has always a negative Value:

So that we can only diminish an Object, and make it appear less, by a convex Speculum, and that only when the Focus is affirmative, or the Image to be represented behind the Speculum. And by a concave Speculum there is no diminishing an Object, as long as the Focus is affirmative; for b being greater than c , the Rule

in that Case $d = \frac{cr - br}{2c}$ will have a negative Value:

So that we can only magnify an Object, and make it appear greater behind a concave Speculum. But if the Focus be negative, and the Image and Object to be

be both on the same Side of the Speculum,

$$br + cr$$

the Rule being $d = \frac{br + cr}{2c}$, shews that in

this Case a concave Speculum will magnify or diminish an Object in what Proportion we please.

It is to be observed, that if the Object be a right Line, the Proportion b to c will express the Proportion between the Object and Image themselves; but if the Object be a plane Figure, the Proportion b to c will be only subduplicate of that which the Object bears to the Image, as we learn from *Euclid*. So that if b to c be as 2 to 1, the Object and Image will be as the Squares of these Numbers, or as 4 to 1. But it must also be noted, that Painters usually measure the Largeness of their Figures by the simple Proportion of their homologous Lines; so that when they speak of an human Figure twice as big as the Life, their Meaning is, that the homologous Lines of this Figure are twice as great as those of the Life; or that the Dimensions of every Member in Length and Breadth are twice as large as those of the Men represented; though properly speaking, the Picture is four times as big as the Life.

If the Out-Lines of the Image be desired twice as big as the Life, and the *Focus* affirmative; c being in this Case greater than

R

b , the

b , the Problem will be impossible for any convex Speculum; but the Object being placed before a concave Speculum at the Di-

stance $\frac{cr - br}{2c} = \frac{r}{4}$, or $\frac{1}{4}$ of the Radius,

will have its Image magnified in the Proportion assigned. If we would have the *Focus* negative, and the Image represented on the same Side of the Speculum with the Object, still the Problem will be impossible for a convex Speculum; but if the Object be placed before a concave Speculum

at the Distance $\frac{br + cr}{2c} = \frac{3r}{4}$, or $\frac{3}{4}$ of the

Radius, its Image made by the Speculum will be magnified in the Proportion assigned.

Thus let the Proportion b to c be what it will, the Rule will always give us the Distance, at which the Object must be placed before the given Speculum, in order to have its Image magnified or diminished in that Proportion. I shall only add one Instance more; and that is, supposing b and c were equal, and the *Focus* affirmative: In this Case we shall have both for convex and

$$\text{concave Specula } d = \frac{+br - cr}{2c} = \frac{0}{2c} = 0:$$

That

That is, the Object must be placed at the very Vertex of the Speculum; in which Case we know both Object and Image coincide. If the *Focus* were negative, then no convex Speculum will do; and the Rule for *concave Specula* will be

$$d = \frac{br - cr}{2c} = \frac{2cr}{2c} = r: \text{That is, the Ob-}$$

ject must be placed in the Center of the Speculum.

There are two farther *Uses* to be made of this Method; the first is, the Distance at which the Object is to be placed before the Speculum, and the Proportion in which the Image is to be magnified or diminished, being given, to find what Degree of Convexity or Concavity the Speculum should have, in order to magnify or diminish the Image in the Proportion assigned: That is,

$$\text{in the Rule } d = \frac{b - cr}{2c},$$

given, we are required to find r ; which will give us, in the Case of a convex Spe-

$$\text{culum, and an affirmative } Focus, r = \frac{2cd}{b - c}$$

and in the Case of a concave Speculum,

$$r = \frac{2cd}{c - b}. \text{ If the } Focus \text{ be negative, for}$$

convex *Specula*, the Rule stands $r = \frac{2cd}{b-c}$;

for concave, $r = \frac{2cd}{b+c}$.

From all which it appears, that if c be greater than b , or if the Image be desired greater than the Object, and to be represented behind the Speculum, no Convex will do, but a Concave will, whose Radius

is $\frac{2cd}{c-b}$. As likewise, if b be greater than

c , or the Image be desired less than the Object, and to be represented behind the Speculum, no Concave will do, but a Convex

will, whose Radius must be $\frac{2cd}{b-c}$. If the

Focus be required negative, or the Image to be on the same Side of the Speculum with the Object; the Problem is altogether impossible for a convex Speculum, whether to magnify or diminish; and always possible for a concave Speculum either to magnify or diminish.

The other *Use* we may make of this Method is, from the Distance of the Object before the Speculum, and the Radius of Convexity or Concavity being given, to find the Proportion b to c , which the Object will

will bear to its Image made by the given Speculum: That is, in the forementioned Rule, having d and r given, to find the Proportion of b to c . Whence in the Case of a convex Speculum, and an affirmative Focus, 'twill be $b . c :: 2d + rr$; and in the Case of a Concave, $b . c :: r - 2d . r$. But if the Focus be negative, in *convex Specula*, 'twill be $b . c :: - 2d - r . r$; and in *concave*, $b . c :: 2d - r . r$.

So then in the Case of a convex Speculum, and the Image behind the Speculum, b will always be greater than c ; because $2d + r$ is of necessity greater than r : And in the Case of a Concave, b will always be less than c for the like Reason; and if d be equal to $\frac{1}{2} r$, then c will be infinite in respect to b . But if the Image be required on the same Side of the Speculum with the Object, in *convex Specula*, 'twill be found always impossible; and in *concave*, possible in all Cases whatever, both of magnifying and diminishing: For if d be greater than r , then b will be greater than c ; if d be less than r , b will be less than c ; and if d be equal to $\frac{1}{2} r$, then c will be infinitely greater than b .

III. At Corol. 3. Prop. XV.

THE Author gives a Construction to find the Focus of Rays refracted at a Spherical Surface; and towards the latter end of that Corol. applies the same to the Case of a plane Surface: Affirming, that in this Case the right Line FM , which determines the Focus by its meeting the Axis BA produced somewhere in C , will be the same with a right Line, joining the Point F with another Point taken in BE produced, at such a Distance from B , as to make that Distance bear the same Proportion to BE which AN does to AM ; because the Tangents of Angles are reciprocally as their Cotangents.

[Plate III. Fig. 5.] That this may be more easily conceived; Suppose the Surface BD plane, and take the Point C upon BE produced; so as that BC may be to BE as AN to AM , and join FC . We are to shew that the right Line FM produced, will in this Case meet the Axis AB produced, in the Point C , and there determine the Focus. Join EN , EM ; and on the Center E with any Radius, as EA , describe an Arch of a Circle AG , and draw the Tangents of the Angles AEN , AEM , which will alway be as AN to AM , and
are

are in the present Case those very Lines, as likewise their respective Cotangents GH , GK . Since A , the Center of the plane Surface BD , and consequently the right Line AMN , is at an infinite Distance from B , the right Line FM becomes parallel to EM ; and consequently if produced beyond F , will meet the Axis AB produced somewhere, suppose at C , so as to make the Triangle CBF similar to the Triangle KGE . And therefore the right Line EH , passing thro' the Vertex of both those Triangles, will cut their Bases CB , GK similarly in E and H , so as to make BC to BE , as GK to GH . But because the Tangents of Angles are reciprocally as their Cotangents, AN is to AM , as GK to GH , therefore BC is to BE , as AN to AM . And consequently the Point C , where the right Line FM produced meets the Axis AB produced, is that very Point C taken at first upon BE produced, so as that BC may be to BE , as AN to AM . Q. E. D.

IV. At Prop. XVI. Prob XI.

THE Author recommends the Use of analytical Calculations, for finding the Foci of Lenses, as far better than the very nicest geometrical Constructions: For
which

which Reason it cannot be improper to give the less skilful Reader an Example, that he may see how such Calculations are to be managed.

[Plate III. Fig. 6.] Suppose BD to be the given *Lens*, and E a Point in its Axis, from which Rays diverging fall upon the *Lens*, A the Center of its first spherical Surface, and C the Center of its second spherical Surface, BD the Thickness of the *Lens*, and I to R the *Ratio* of Incidence to Refraction. And it is required to find F the *Focus* of those Rays after Refraction at both Surfaces of the *Lens*. We must first find *f* the *Focus* of those Rays after their Refraction at the first Surface only, or their *Focus* of Transition. Call EB *d*, BD *t*, AB *r*, CD *s*, Bf *x*, DF *y*. By *Proposit.* XV. EA (*d* + *r*) : Af (*x* - *r*) + fB (*x*) : BE (*d*) :: I : R : Whence multiplying the Extremes and Means, $Rd x + R r x = I d x -$
 $I d r$

$$I d r; \text{ and } B f (x) = \frac{I d - R d - R r}{I d - R d - R r}.$$

Whence it is plain, that if *d* be so great in respect to *r*, that *I d* exceeds *R d* + *R r*, the *Focus f* is affirmative, and to be taken from B the Vertex of the refracting Surface directly forward, as at Fig. 6 : If *I d* is less than *R d* + *R r*, the *Focus f* is negative, and to be taken from B backwards, as at

Fig.

Fig. 7; and if $I d$ be equal to $R d + R r$, $B f$ becomes infinite, and the Rays proceed parallel.

Thus having found f the *Focus*, after the first Refraction, we may, by the same means, find F the *Focus* after the second Refraction. For, by the same

$$\text{Prop. XV. } f C \left(\frac{I d r}{I d - R d - R r} - t + s = \frac{I d r - I d t + R d t + R r t + I d_e - R d_e - R r_p}{I d - R d - R r} \right) :$$

$$C F (y + s) + F D (y) : D f \left(\frac{I d r}{I d - R d - R r} - t = \frac{I d r - I d t + R d t + R r t}{I d - R d - R r} \right) ::$$

$R : I$: (I to R at the Emerfion of Rays from any Lens, being as R was to I at their Immersion into it.) Wherefore multiply-

ing the Extremes and Means, we have $I^2 d r y - I^2 d t y + I R d t y + I R r t y + I^2 d_e y - I R d_e y - I R r_e y = I R d r y - I R d t y + R^2 d t y + R^2 r t y + I R d r_p - I R d t_p + R^2 d t_p + R^2 r t_p$: And confequently, $D F (y) =$

$$\frac{I R d r_p - I R d t_p + R^2 d t_p + R^2 r t_p}{I^2 d r - I R d r - I^2 d t + 2 I R d t - R^2 d t + I R r t - R^2 r t + I^2 d_p - I R d_e - I R r_p.}$$

$$\frac{R}{I - R},$$

Which Equation, if we put $p = \frac{R}{I - R}$, may be abridged, and reduced to $D F (y) =$

$$\frac{I p d r_p - R d p t + R p r p t}{I d r - I d t + R r t + I d_p + R d t - I p r_p.}$$

S

And

And it is evidently the same with that, which the famous Dr. *Halley* has given long ago, in the *Philosophical Transactions*, for finding the Foci of optical Glasses universally.

This Calculation being general, will serve for all Sorts of *Lenses*, be the Matter of which they are made, and the ambient *Medium* what they will, or whatever be the *Ratio* of I to R: And tho' it is made for *Lenses* whose Surfaces are both *convex*, yet, *mutatis mutandis*, it will extend to *Lenses* of any other Figure whatever, whether *double-convex* or *double-concave*, *plano-convex* or *plano-concave*, or *convexo-concave*, which last Sort are commonly called *Menisci*. For the Radius of a *concave* Surface being on the contrary Side, or *negative* with respect to that of a *convex*, and the Radius of a *plane* Surface *infinite*; 'tis only changing all the Signs + or — with which the Radius of the respective Surface, which we would have *concave* instead of *convex*, is affected in the general Rule; or making all the Terms *infinite*, which involve the Radius of the respective Surface, which we would have *plane* instead of *convex*. So likewise if we would have it extend to other Rays besides *diverging* ones; the Distance of the Point where *converging* Rays meet, from the first Surface of the *Lens*, being on the contrary Side or *negative*, in respect to that of *diverging* Rays, and the Distance of the
Point

Point where *parallel* Rays meet, from the same Surface, being *infinite*: 'Tis only changing the Signs of all the Terms where we meet with d , if the Rays are supposed *converging*; or making those same Terms infinite if the Rays are supposed *parallel*.

In the Case of a *double Convex* of Glafs, if the ambient *Medium* be Air, I being to R as 3 to 2, we shall have the *focal* Distance from the second Surface of the *Lens*,

$$6dr_s - 2d_{pt} + 4r_{pt}$$

$$y = \frac{6dr_s - 2d_{pt} + 4r_{pt}}{3dr - dt + 2rt + 3d_p - 6r_p} : \text{If the}$$

ambient *Medium* be Water, I being to R as 9 to 8, the Rule will be

$$72dr_s - 8d_{pt} + 64r_{pt}$$

$$y = \frac{72dr_s - 8d_{pt} + 64r_{pt}}{9dr - dt + 8rt + 9d_p - 72r_p}. \text{ For a}$$

double Convex of Water, and the ambient *Medium* of Air, I being to R as 4 to 3, the

$$12dr_s - 3d_{pt} + 9r_{pt}$$

$$\text{Rule is } y = \frac{12dr_s - 3d_{pt} + 9r_{pt}}{4dr - dt + 3rt + 4d_p - 12r_p}.$$

And for a *double Convex* of Diamond, in a *Medium* of Air, I being to R as 5 to 2, the Rule

$$\frac{1}{3}dr_s - 2d_{pt} + \frac{4}{3}r_{pt}$$

$$\text{would be } y = \frac{\frac{1}{3}dr_s - 2d_{pt} + \frac{4}{3}r_{pt}}{5dr - 3dt + 2rt + 5d_p - \frac{1}{3}r_p}.$$

If the Thickness of the *Lens* be neglected, which is generally not considerable; the Terms where t occurs being rejected, the Rule is still farther abridged to

$$y = \frac{pdrp}{dr + dp - prp}.$$

Where it is evident, that if d be so small in respect to r and p , that $dr + dp$ is less than prp , the focal Distance

$$y \text{ will be negative, and } = \frac{pdrp}{-dr - dp + prp},$$

or the Rays after the two Refractions at both Surfaces of the *Lens*, will still proceed diverging from some Point, before the second Surface of the *Lens*; and if $dr + dp$ be equal to prp , y is infinite, and the Rays after Emerision from the *Lens* proceed parallel. The Error in neglecting t is so small, that if, for the Ease of the Calculation, we suppose a *Lens* of Glass equally *convex* on both Sides, and exposed to *parallel Rays*, r being in this Case equal to p , and d infinite, the focal Distance, when t is neglected, is

$$\frac{2dr}{2dr} = r, \text{ and when it is considered}$$

$$\frac{6rr - 2rt}{6r - t}; \text{ which is only } \frac{rt}{6r - t}, \text{ or nearly}$$

$\frac{1}{6}t$ less than the former. In the Case of

converging Rays falling upon a *double Con-*

$$\text{vex of Glass, we have } y = \frac{-2drp}{-dr - dp - 2rp},$$

always

always affirmative : And if the Rays are *parallel*, d being infinite, 'twill be

$$y = \frac{2dr_e}{dr + d_e} = \frac{2r_e}{r + e}; \text{ which also gives the}$$

Focus always affirmative, or behind the *Lens*. *Diverging Rays* falling upon a

double Concave, give $y = \frac{2dr_e}{-dr - d_e - 2r_e}$

always negative, as in the Case of *converging Rays* on a *double Convex* 'twas always affirmative : But if the Rays are *converging*,

it will be $y = \frac{-2dr_e}{dr + d_e - 2r_e}$ affirmative,

when $dr + d_e$ is less than $2r_e$, or when the *Focus* of *diverging Rays* collected by a *double Convex* is negative, and *vice versa* : If the Rays are *parallel*, 'tis

$$y = \frac{2dr_e}{-dr - d_e} = \frac{2r_e}{-r - e}, \text{ always nega-}$$

tive. A *plano-convex* Glass, the *plane* Surface being exposed to *diverging Rays*, gives,

$$r \text{ being infinite, } y = \frac{2dr_e}{dr - 2r_e} = \frac{2d_e}{d - 2e}, \text{ af-}$$

firmative or negative, according as d is greater or less than $2e$; if exposed to *converging*

Rays, $y = \frac{-2d_e}{-d - 2e}$, always affirmative ;

if

if to *parallel* Rays, $y = \frac{2d_p}{d} = 2s$: So that

the Image of a vastly distant Object is always formed by a *plano-convex Lens*, the plane Side being turned towards the Object, just at the Distance of the Diameter of the second Surface behind it. A *plano-concave Lens*, exposed on the *plane* Side to diver-

ging Rays, gives $y = \frac{-2dr_p}{dr + 2r_s} = \frac{-2d_p}{d + 2s}$,

always negative; to converging Rays,

$y = \frac{2d_s}{-d + 2s}$, affirmative, when the Fo-

cus of diverging Rays on a *plano-convex* is negative, and *vice versa*; to *parallel* Rays,

$y = \frac{-2d_p}{d} = -2s$: So that the Image of

a vastly distant Object is always formed by a *plano-concave* at the Distance of the Diameter before the second Surface, as it is by a *plano-convex* behind. A *Meniscus* exposed on the *concave* Side to *diverging* Rays, gives

$y = \frac{-2dr_p}{-dr + d_p + 2r_p}$, affirmative only

when d and r are so great in respect to s , that dr exceeds $d_s + 2r_s$: To *converging*

Rays, $y = \frac{2dr_s}{dr - d_s + 2r_s}$, affirmative or

negative,

negative, according as $dr + 2r\rho$ is greater or less than de : If to *parallel* Rays

$$y = \frac{-2dr\rho}{-dr + d\rho} = \frac{-2r\rho}{-r + \rho}, \text{ affirmative,}$$

if the Radius of *Concavity* is greater than the Radius of *Convexity*; and negative if less; and infinite if equal: For the Effects of the first Surface are, in that Case, exactly destroyed by the second, and the Rays suffered to proceed still parallel.

It is to be observed, that if the Thickness of the *Lens* is neglected, as inconsiderable, the *Focus* of all Sorts of Rays falling upon any *Lens* will be exactly the same, upon whichever Surface of the *Lens* they are first received. But if the Thickness of the *Lens* be considered, there will be some Difference in the *focal* Distance, according as you turn this or that Surface of the *Lens* towards the Rays. And this Difference is easily found from the general Rule: For upon turning the other Surface of the *Lens* towards the Rays, ρ becomes r , and r changes to ρ ; by which means the Rule will give us the *focal* Distance in both Cases, and subtracting one from the other, we find their Difference. Thus, if, to abridge the Rule, we suppose the Rays parallel, d being infinite, we have for a *double Convex*

$$\text{in one Case } y = \frac{I\rho r\rho - R\rho t}{I\rho - I\rho t + I\rho + R\rho t}, \text{ and upon}$$

turning

turning the *Lens*, $y = \frac{I_{per} - R_{rt}}{I_r - I_t + I_r + R_t}$;

wherefore subtracting one from the other, according as p or r is greatest, we shall have the Difference in *double Convex*, occasioned

by turning the *Lens* $= \frac{+ R_{rt} - R_{et}}{I_r - I_t + R_t + I_p}$,
 $\pm 2rt \mp 2et$

or in *Glass* $\frac{3r - t + 3p}{3r - t + 3p}$. And this is ap-

plicable to *Lenses* of any other Figure, by changing the Signs $+$ or $-$ of those Terms, where we meet with r or p , or making them infinite, according as the respective Surfaces are *concave* or *plane*. Thus in the Case of a *plano-convex*, r being infinite, the Difference arising upon turning the *Lens*

$\frac{R_{rt}}{I_r} - \frac{R}{I} = -t$; or in *Glass* $\frac{2}{3}t$, in

Water $\frac{3}{4}t$, and $\frac{2}{5}t$ in *Diamond*. Which shews that the *focal* Distance is greater by $\frac{2}{3}t$ when the *plane* Side of a *plano-convex* of *Glass* is turned towards a *vastly distant* Object, than when the *convex* Side is turned to it. After the same manner, the negative *focal* Distance of a *plane Concave* will be greater by $\frac{2}{3}t$, when the *plane* Side is turned towards the *vastly distant* Object, than when the *concave* Side is turned to it. In *double Con-*
caves,

caves, where the *Focus* is always negative,

$$\pm 2pt \mp 2rt$$

the Difference is —————, according

$$- 3r - t - 3p$$

as p is greater or less than r . In *Menisci*, the *focal* Distance, whether affirmative or negative; being always greatest when the *concave* Surface is turned towards a vastly distant Object, the Difference is

$$- 2rt - 2pt$$

————— when the *Foci* upon turning

$$- 3r - t + 3p$$

either Side fall both one way, and are either both affirmative or negative: But if t be so considerable as to be greater than $3p$, the *focal* Distance, upon turning the *con-*

$$- 6rp - 2pt$$

cave Side, ————— is affirmative, and

$$- 3r - t + 3p$$

the *focal* Distance, upon turning the *convex*

$$- 6pr + 2rt$$

Side, ————— negative; and conse-

$$3p - t - 3r$$

quently their Difference is —————

$$2rt - 12rp - 2pt$$

$$- 3r - t + 3p :$$

And if t were equal to $3p$, the *Focus* in this Case, upon turning the *convex* Side to a vastly distant Object, falling exactly upon the *Vertex* of the second Surface of the *Lens*, and consequently the *focal* Distance being equal to nothing, the Difference will be the

T

same

same with the *focal* Distance upon turning

$$-6r_s - 6s_s.$$

the *concave* Side, namely, ——— After

$$- 3r$$

the like manner, may be found from the general Rule, the Difference which would arise upon turning the different Surface of any Sort of *Lens* towards other Rays besides *parallel*, whether *diverging* or *converging*; but the *Canons* for these Cases consist of so many Terms, and are of so little Use, that they are not worth having.

There are three farther *Uses* to be made of the general Rule above delivered; the first is, from the *Lens* or *Focus*, where an Object is represented, being given, to determine the *Distance* of the Object from the *Lens*; or the *Lens* by which we would form the Image of any Object, and the *Focus* where we would have it formed, being given, to determine the Distance at which the Object should be placed before the *Lens*, that it may be represented in the given *Focus*:

$$pdre$$

That is, in the Equation $y = \frac{pdre}{dr + ds - prp}$,

$$dr + ds - prp,$$

or $dry + d_y - pr_y = pdre$, r , s , p , and y being given, 'tis required to find d , and con-

$$pr_y$$

sequently we shall have $d = \frac{pr_y}{ry + sy - prp}$;

$$ry + sy - prp;$$

where 'tis plain, that if r and s be so great
in

in respect to y , that pre exceeds $ry + ey$, d will be negative; and the Object cannot be represented in the Circumstances required, unless by means of another *Lens*, we first make the Rays coming from the Object, *diverging* converge to a Point behind the first Surface of the *Lens* given, at the Distance of

$$\frac{prey}{pre - ry - ey}$$

— : And if pre is equal to $ry + ey$,

$$pre - ry - ey$$

d will be infinite. Suppose the given *Lens* a *double Convex* of Glass, and made of two Segments of equal Spheres, but of a Thickness not considerable, and it is required to find at what Distance from the *Lens* a lucid Body should be placed, in order to have its Beams parallel after their Emerfion from the *Lens*, and consequently its Light thrown upon Objects *vastly distant*, which may be thereby illuminated: In this Case y being infinite, and r equal to ρ , and p equal to 2, we shall have the Distance required $d = r$. But if t be considerable, we must find d from the Rule which takes in the Thickness of the *Lens*, which gives us the exact Value of

$$4ret + 64ey - 2rty$$

$d = \frac{4ret + 64ey - 2rty}{3ry - ty + 3ey - 6rp + 2et}$: As if,

$$3ry - ty + 3ey - 6rp + 2et$$

for Example, the *double Convex* just mentioned, were an entire *Sphere* of Glass, and the same thing required as before, y being, as we have already observed, infinite,

and r equal to e , and moreover t equal to $2r$;

this last Rule gives $d = \frac{6rry - 4rry}{3ry - 2ry + 3ry} =$

$\frac{1}{2}r$; whereas by the former, which neglects

the Thickness, we have the Distance required twice as great, or $d = r$; a Difference very considerable, if the Spheres be of any Bigness. So then, a lucid Body placed at the Distance of half the Radius from a Sphere of Glass, or at the Distance of the whole Radius from a *double Convex* of equal Spheres, whose Thickness is inconsiderable, will illuminate Objects vastly distant. If the given *Lens* were a *Hemisphere* of Glass, and the same thing still required; if the *convex* Surface be first, both y and e being in this Case infinite, and t equal to r , the Distance of the lucid Body will be

$d = \frac{6rey}{3ry} = 2r$; but if the *plane* Side be

next the lucid Body, y and r being infinite,

we have $d = \frac{6rpy - 2rty}{3ry} = \frac{4}{3}e$, or $\frac{4}{3}r$; there

being as has been shewn before a Difference of $\frac{2}{3}t$ in the *focal* Distance of a *plano-convex* exposed to *parallel* Rays, occasioned by turning the different Sides of the *Lens*: If t had been

been neglected, we should have had $d = 2r$ in both Cases. If we have an Object represented by a *double Concave* of Glass of equal Spheres, at a negative *Focus* the Distance of the Radius from the *Lens*, and it were required to find the Distance of the Object, y , r , and e being all negative and equal, and t inconsiderable, we shall have

$$d = \frac{-2rrr}{2rr - 2rr} \text{ infinite; and consequently}$$

the *Object* is *vastly distant*. The same thing may be done for all other Cases whatever, only remembering to make the proper Alterations according as r , e , or y are negative or infinite, and t considerable or inconsiderable.

The second *Use* is, from one Surface (either the first or second) of a *Lens* being given already formed, to find what Degree of *Convexity* the other Surface must have, in order to represent a given Object at a given *Focus*: That is, in the Equation before used $dry + dey - prey = pdr$, d , y , p and r being given, to find e , or e being given, to find r . Whence we have

$$e = \frac{dry}{pdr - dy + pry}, \text{ and } r = \frac{dey}{pde - dy + pey};$$

which will serve for any other *Lenses* besides *double-convex*, and any other Rays besides *diverging* ones, by making such Alterations

terations as have been already directed. If the first Surface of a Glass *Lens* were *plane*, and it were required to find what Degree of Convexity the second Surface must have, in order to represent an Object at a *Focus* just as far distant from the *Lens* as the Object it self; in this Case d is equal to y , and

$$r \text{ infinite, and consequently } e = \frac{dry}{2pry} = -\frac{1}{4}d:$$

So that the second Surface of the *Lens* must be made of a Segment of a Sphere, whose Radius is equal to $\frac{1}{4}$ of the Distance of the Object. If the Object to be represented at a given *Focus*, be *vastly distant*, d being in this Case infinite, the Rule is abridged to

$$e = \frac{ry}{pr-y} : \text{Whence 'tis plain, that if in}$$

Glass y is greater than $2r$, or the given *Focus* be at a greater Distance from the *Lens* than twice the Radius of the given Surface, e will be negative, and the second Surface must be made *concave*; and if y be equal to $2r$, e is infinite, and the second Surface must be *plane*. If the Thickness of the *Lens* be so great that it ought to be considered, we must find e from the general Rule.

The third *Use* is, from the *Lens*, *Distance* of the Object, and *Focus* being given to determine the *Ratio of Refraction*: That is, in the Equation before used $dry + dey - prsy = pdre,$

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 $= pdre, d, r, e$ and y being given, to find
 p , which gives us $p = \frac{dry + dey}{dre + rey}$: For p

being found, the Value of $p = \frac{I}{I - R}$ gives
the *Ratio* I to R as $I + p$ to p . If a *double-convex Lens*, made of two Segments of the
same Sphere, represents, or is required to
represent, a vastly distant Object, at a *Focus*
the Distance $\frac{3}{2}r$ from the *Lens*; d being in
this Case infinite, and r equal to e , and y
equal to $\frac{3}{2}r$, we shall have $p = \frac{3drr}{drr} = 3$,

and consequently the *Ratio* I to R as 4 to 3;
whence the *Lens* is made, or ought to be
made, of the *Medium* of Water. If the *focal*
Distance were $\frac{4}{3}r$, we should have $p = \frac{2}{3}$, or
I to R as 5 to 2, and consequently the *Lens*
would be *Diamond*. If the *focal* Distance
were $4r$, then we have $p = 8$, or I to R as 9
to 8, and the *Lens* is *Glass*, and the *ambient*
Medium Water. But if we are very curious
in determining the *Ratio* of *Refraction*, it is
done more exactly when the *Lens* is formed
into an *Hemisphere*, or a *plano-convex*, and
receiving the Rays of the Sun upon its *plane*
Side, collects them in a Point at some Di-
stance behind, which must be measured
with great Niceness; because in this Case
our neglecting t occasions no Error at all.
In

In this Case, if the *focal* Distance is equal to thrice the Radius of the Sphere, d and r being infinite, and y equal to $3e$, 'twill be $p = 3$, or I to R as 4 to 3; if y is equal to $2e$, I is to R as 3 to 2; and if y is equal to e , then I is to R as 2 to 1.

V. At Prop. XX. Prob. XIII.

WHich is, *To find the Distance at which an Object should be placed from a given Lens, so as that the Image formed by the Lens may bear a given Proportion to the Object.*

The Author has given the *Construction*, but omitted the *Demonstration*; leaving the Process of the Calculation which points out that Construction, as a Trial of Skill to the diligent Reader. But because my Design in publishing the Book is to make it entirely easy; for fear it may prove too difficult or discouraging a Task to some who are either not skilful enough, or perhaps too lazy, to go through with it, I have subjoined the following *Solution*.

[*Plate III. Fig. 6, 7.*] Let the given *Lens* BD be a *double Convex*, and call AB the Radius of the first Surface a , CD the Radius of the second Surface b , BD the Thickness of

of

of the *Lens* c; the *Proportion* of the *homologous* Lines of the Object and Image as r to b , and $E B$ the Distance required z , at which the Object is to be placed before the *Lens* B, which we suppose made of Glass, and the ambient *Medium* Air. It is plain there are different Values of $E B (z)$ according as the *Focus* F is affirmative, or beyond the *Lens*, as at *Fig. 6.* or negative, and on the same Side of the Object, as at *Fig. 7.* Both which Cases shall be respectively considered, and included in the Demonstration.

By *Corol. Prop. XVIII.* the *homologous* Lines of the Object and Image are to another as their respective Distances from the *Lens*; wherefore r is to b as $E B (z)$ the Distance of the Object required to $D F$ the *focal* Distance of the desired Image, which is consequently $\frac{bz}{r}$. But we shall have an-

other Value of this *focal* Distance from *Prop. XV.* For if we look upon f as the *Focus* of Rays sent diverging from the Object at E , after their Refraction at the first Surface of the *Lens*, and F their *Focus* after both Refractions; and call the first *focal* Distance $Bf, = x$, and the second $D F, = y$. Before we can find the *focal* Distance $D F$, which determines the Place of the *Image*, we must first find Bf . Now to find Bf , by *Prop.*
U XV.

XV. $\frac{EA}{3} \left(\frac{z+a}{3} \right) : AB(a) :: Ef(\pm z+x) : Bf(x)$; whence $zx + ax = \pm 3az + 3ax$, and $Bf(x) = \frac{\pm 3az}{z-2a}$. Having found Bf

the same Proposition gives us DF, for $\frac{fC}{2} \left(\frac{\pm 3az}{z-2a} \mp c \pm b \right) =$

$$\frac{\pm 3az \mp cz \pm bz \pm 2ac \mp 2ab}{2z - 4a} : CD$$

$$(b) :: fF \left(\frac{3az}{z-2a} - c \pm y = \frac{3az - cz \mp yz \pm 2ac \pm 2ay}{z-2a} \right) : DF$$

$$(y); \text{ whence } \pm 3azy \pm czy \pm bzy \pm 2acy \mp 2aby = 6abz - 2bcz \mp 2bzy + 4abc \pm 4aby, \text{ and } DF (=y) = \frac{6abz - 2bcz + 4abc}{\pm 3az + cz \pm 3bz \pm 2ac \mp 6ab}.$$

Therefore comparing the two Values of DF together, we have this Equation $\frac{bz}{r} =$

$$\frac{6abz - 2bcz + 4abc}{\pm 3az + cz \pm 3bz \pm 2ac \mp 6ab}, \text{ or } \pm 3abz^2 \mp bcz^2 \pm 3bz^2 \pm 2abcz \mp 6abz^2 = 6abrz - 2bcrz + 4abcr.$$

From whence we have $zx = \frac{6abz - 2acz \pm 6arz \mp 2rcz \pm 4arc}{3a - c + 3b}$:

$$3a - c + 3b$$

And

And if the Thickness of the *Lens* be neglected as inconsiderable, all the Terms where C occurs vanishing, we have $z z = \frac{6abz + 6arz}{3a + 3b}$, and consequently $EB(z) = \frac{2ab + 2ar}{a + b}$. Q. E. I.

[Plate II. Fig. 22.] If then we would *construct* this Equation, we have this Proportion given us for that Purpose, $a + b : b + r :: 2a : z$. Taking therefore A B equal to a , and B C in the same right Line equal to b ; from C draw at pleasure the indefinite right Line C M, upon which cut off C D equal to C B (b), and from D on either Side take D M, or D m such, that it may bear the same *Proportion* to D C, which the *homologous* Lines of the Object do to those of the Image. Join A M, or A m , to which thro' B draw B E, or B e , parallel; and twice M E, or twice $m e$, is the *Distance* required in the *Problem*. For A C ($a + b$): M C, or $m C$ ($b + r$) :: 2 A B ($2a$) 2 M E or 2 $m e$ (z). And consequently the *Construction* gives the true Value of z , as before found. Q. E. D.

The same *Problem*, of magnifying or diminishing a given Object by a given Lens, in any assigned Proportion, may be solved from the Equation above given, for finding the *Foci* of all sorts of *Lenses*, which if ex-

pressed in the Characteristicks in present

$$pabz$$

Use, is $y = \frac{pabz}{az + bz - pab}$. For supposing as

before, r to b expresses the Proportion which the *homologous* Lines of the Object are required to bear to those of the Image, and 'tis desired to find z the Distance of the Object from the *Lens*, which is necessary to perform the Conditions required: By *Corol. Prop. XVIII.* we have another Value of the *focal* Distance $y = \frac{bz}{r}$. Whence comparing

both together we have $\frac{bz}{r} = \frac{pabz}{az + bz - pab}$,

and consequently if the *Focus* is to be affirmative $z = \frac{pab - par}{a + b}$; but if pab be great-

er than $az + bz$, then the *Focus* is negative, or on the same Side with the Object, and this negative *focal* Distance is

$\frac{pabz}{-az - bz + pab}$, and consequently $z = \frac{pab - par}{a + b}$: That is, if the *Lens* be Glass,

the following Equation includes both Cases, whether the Image is to be represented on the contrary or same Side with the Object,

$z = \frac{2ab - 2ar}{a + b}$. Where it is to be observed,

that

that if the Image is represented on the contrary Side by a *double-convex Lens*, or at an affirmative *Focus*, it may be made either equal to, greater or less than the Object in what Proportion we please; but if it is represented on the same Side, or at a negative *Focus*, r must always be less than b , and consequently the Image may be shewn larger than the Object in all the Degrees imaginable, but never less, nor equal; for when b is equal to r , and the *Focus* negative, z is $= 0$, and when b is less than r , z is negative and impossible.

Nor is this Solution confined to the Case of *double Convexes* only, tho' made for *Lenses* of that Figure, but will with proper Alterations extend equally to *Lenses* of all other sorts whatever; only observing to change the Signs $+$ or $-$ with which the Radius of a *concave* Surface is affected, or making the Terms infinite where the Radius of a *plane* Surface occurs; because it has been shewn before, that the Radius of a *concave* Surface bears a contrary Sign to that of a *convex*, and the Radius of a *plane* Surface is infinite. If then the *Lens* be a *concavo-convex*, or *Meniscus* of Glass, and the first Surface concave, the Rule for both Cases, whether the *Focus* is to be affirmative or

negative, is $z = \frac{-2ab \pm 2ar}{-a + b}$; and if the

second

second Surface be that which is *concave*, then for the affirmative and negative *Foci* respectively, the Rule becomes $z = \frac{-2ab \pm 2ar}{a - b}$. Where it is to be observed,

that in the first Case if the *Focus* be affirmative, a must be greater than b , or else z will be negative, and the Problem impossible; and if the *Focus* be negative, and b greater than r , then must a be still greater than b ; but if b be less than r , then a must be less than b , or else the Problem will be impossible: And in the last Case, if the *Focus* be affirmative, and b greater than r , then b must be greater than a , and *vice versa*; and if the *Focus* be negative, b must still be greater than a . Which shews that in the Case of a *Meniscus* turned on the *concave* Side towards the Object, the Image can never be represented at all on the opposite Side, unless the *concave* Surface be a Segment of a larger Sphere than the *convex*, and then it may be shewn in what Proportion to the Object we please; and if it is to be represented on the same Side with the Object, and magnified, the Radius of *Concavity* must be still larger than that of *Convexity*, and *vice versa*, if the Image is to appear diminished. And in like manner may be understood what will happen upon turning the convex Side of the *Meniscus* towards

wards the Object. If the *Lens* be a *double concave* of Glass, the *Focus* being in this Case always negative, we have but one Value of z , which is $\frac{2ab - 2ar}{-a - b}$, affirmative only when r is greater than b ; which shews that a *double-concave* can only diminish. If the *Lens* be a *plano-convex* of Glass, it will be $z = 2b \pm 2r$; which shews that in this Case the Object may either be magnified or diminished, if the *Focus* be affirmative, but only magnified if it be negative. It must be noted, that if the second Surface, whose Radius is b , were supposed plane, we should have $z = 2a$; because b not only stands for the Radius of that Surface, but also expresses the *Ratio* of the Image to the Object, which is consequently in this Case infinite, and the Image *vastly distant*. If the *Lens* be a *plano-concave* of Glass, the *Focus* being always negative, we have only one Value of z , which is $z = -2b \pm 2r$; which shews, that a *plano-concave* can only diminish.

It may be remarked, that those Cases which make the Distance z negative, and the Problem impossible for the given *Lens*, may by means of another *Lens* be made practicable: If we first receive the Rays of the Object upon this second *Lens*, and before

fore they are collected at the *Focus* throw them upon the given *Lens*, in such a manner as to make them fall *converging* to a Point behind the first Surface of the given *Lens*, at the Distance of the negative Value of z .

If the *Lens* were a *double-convex* of *Water*, p being $= \frac{R}{1 - R}$, it is $z = \frac{3ab \pm 3ar}{a + b}$;

if of *Diamond* $z = \frac{\frac{2}{3}ab \pm \frac{2}{3}ar}{a + b}$; if of *Glass*

in an ambient *Medium* of *Water* $z = \frac{8ab \pm 8ar}{a + b}$; if of *Diamond* in an ambient

Medium of *Water* $z = \frac{\frac{2}{7}ab \pm \frac{2}{7}ar}{a + b}$, and conformably in all other Cases.

If the Image be desired just as great as the Object, then for a *double-convex* of *Glass*, r being equal to b , the Rule gives the Distance required $z = \frac{4ab}{a + b}$; where it is

plain the Problem is only possible when the *Focus* is affirmative. If the *Lens* were equally *convex*, b being in this Case equal to a , there will only be one Value of z , and that for an affirmative *Focus* $z = 2a$: So that if the Object be placed at twice the Distance of the Radius from the *Lens*, the Image formed

formed at its *Focus* will be just as great as the Object. If the *homologous* Lines of the Image were desired twice as large as those of the Object, r being in this Case equal to $\frac{1}{2}b$, for a *double-convex* of Glafs the Rule gives

$$2ab \pm ab$$

$$z = \frac{2ab \pm ab}{a + b}, \text{ and if the } \textit{Lens} \text{ be equally}$$

convex $z = 3a$ or $\frac{1}{2}a$, according as the *Focus* is affirmative or negative, so that an Object placed at the Distance either of $\frac{3}{2}$ or only $\frac{1}{2}$ the Radius from the *Lens*, is represented at a *Focus*, either affirmative or negative, twice as large every way as the Object, or the Image in its whole Content will be four times as large as the Object. If the *homologous* Lines of the Image were desired twice as small as those of the Object, r being in this Case equal to $2b$, the Rule gives for an equally convex *Lens* only one Value of the Distance $z = 3a$, a *double-convex* being only capable of representing an Object diminished, when the *Focus* is affirmative. If the Image be desired an hundred times larger than the Object, or its *homologous* Lines ten times as large, r being in this Case equal $\frac{1}{10}b$, if the *Lens* be equally *convex*, the Rule gives the Distance $z = 1 \frac{1}{10}a$, or $\frac{11}{10}a$, according as the *Focus* is affirmative or negative. And in general if m to n expresses the Proportion which r bears to b , the

$$pab \pm par$$

Rule laid down at first $\frac{pab \pm par}{a + b}$, becomes

$$\frac{pab \pm p \frac{m}{n} ab}{a + b}, \text{ and if the Convexity be equal,}$$

it is $z = \frac{1}{2} pa \pm \frac{1}{2} p \frac{m}{n} a$. From all which it

appears, that with a *Lens* equally convex on both Sides, in order to magnify a given *Object*, the Distance is always something greater than $\frac{1}{2} pa$, when the *Focus* is affirmative, and always less than $\frac{1}{2} pa$ when the *Focus* is negative, unless the *Ratio* of the Image to the Object be infinitely great, and then n being infinite, 'tis $z = \frac{1}{2} pa$: That is, in Glass the Distance of the Object must always exceed the Radius, if the *Focus* is to be affirmative, or fall short of it if negative, and be equal to it when the Image is to be infinitely great, or vastly distant. And in

order to diminish a given Object $\frac{m}{n}$ in this

Case exceeding *Unity*, the Distance, which has but one Case here, is always greater than pa ; and the more z exceeds pa , the more the Object is diminish'd, and *vice versa*, 'till z becoming equal to pa , the Object and Image are likewise equal.

All this is easily observed in that common Experiment of the *Camera obscura*; where the Rays propagated from external Objects are received by a *Lens*, and transmitted into the Room, and do there paint, upon a white Sheet placed at the *focal* Distance

stance of those Rays from the *Lens*, the Images of their respective Objects, in Colours scarce less lively than those of the Objects themselves. And hence this Problem of magnifying or diminishing a given Object, may be of great Use in *Painting*; for by admitting the Image of any Object by means of a *Lens* into a dark Chamber, in what *Ratio* to the Life we please, 'tis easy to hit the Proportion of every Part with great Exactness, which is otherwise but seldom done, especially in such Pieces where the Figures are either much greater or much less than the Life. 'Tis true, these Images that are represented in the *dark Chamber* by a single *Lens* appear inverted, but may be made erect by using a second *Lens* after the following manner: Place the *Object* at such a Distance from the first *Lens* in the Window, that the *Image* formed by that may be just as big as the Life; then beyond the Place of this *Image* fix the given *Lens* at the Distance required z ; this will form an *Image* of the former *Image* which shall be in the *Ratio* assigned, and also erect, but something less lively. What has been above delivered is likewise of Service in the Construction of the *Magick Lantern*, and other optical Machines, where the *Images* of any *Objects* are to be represented monstrously larger or less than the Life.

VI. At Prop. XXIV. Prob. XVII.

WHich is, *To make a concave Speculum of Glass of a given Thickness; the Radius of whose Concavity is also given, in such a manner, that parallel Rays reflected from the first Surface of the Speculum may meet in the same Point of its Axis with those that pass refracted into the Speculum, and are refracted from the second Surface, and again refracted at their Emer-sion from the Speculum.* Or the first Surface of a *concave Speculum* being already formed, to determine of what Sphere 'tis necessary to take a Segment to form the second Surface in such manner, that an *Object vastly distant* may be represented by Reflexion from both Surfaces in one and the same Place, or that the two Images may be united, and consequently be made more lively. The Author has given two several *Constructions* of the *Problem*; the first exact, and the other only near the Truth, but more expeditious: But for Reasons already mentioned he has omitted their *Demonstration*, which is as follows.

[*Plate III. Fig. 8.*] Let AB be the given Radius of the first Surface, and BD the given Thickness of the *Speculum*, and CD the Radius required of the hindmost Surface

face necessary to perform the Conditions of the *Problem*. Suppose the Point f in fD , the Axis of the *Speculum* to be the *Focus* of the *parallel Rays*, after *Refraction* at their Entrance into the first Surface, ϕ their *Focus* after *Reflexion* from the second Surface, and F their *Focus* after *Refraction* again at their Emerision from the *Speculum*; 'tis required that the Point F should be the same with the *Focus* of the same *parallel Rays* after their *Reflexion* from the first Surface. Call the given Radius AB a , the given Thickness BD c , the first *focal* Distance Bf v , the second $D\phi$ x , the last *focal* Distance BF y , and the required Radius of the hindmost Surface CD z . We must find these *focal* Distances one after another, in order to determine the last BF , which must be equal to the *focal* Distance of *parallel Rays* reflected from a *concave Speculum*. For the first then Bf , by *Prop. XIV.* $Bf(v) : fA(v-a) :: I : R ::$ (in Glass) $3 : 2$; whence $Bf(v) = 3a$. And for the second $D\phi$, by *Prop. IV.* $D\phi(x) : \phi C(z-x) :: Df(3a+c) : fC(3a+c-z)$; whence $3ax+cx-zx=3az+cz-3ax-cx$, and consequently $D\phi(x) = \frac{3az+cz}{6a+2c-z}$. Lastly, to determine the *focal* Distance BF after both *Refractions* at the first Surface, and the *Reflexion* from the last, by *Proposit. XV.* the Emerision being out of Glass

$$\text{Glass into Air, } \frac{\phi A}{2} \left(\frac{a+c - \frac{3az - cz}{6a+2c-z}}{2} \right)$$

$$= \frac{6aa + 8ac - 4az + 2cc - 2cz}{12a + 4c - 2z} \Bigg) : AB$$

$$(a) :: \phi F \left(\frac{3az + cz}{6a + 2c - z} - y - c = \frac{3az + 2cz - 6ay - 2cy + zy - 6ac - 2cc}{6a + 2c - z} \right) :$$

$$BF(y); \text{ whence } 6aay + 8acy - 4azy + 2ccy - 2czy = 6aaz + 4acz - 12aay - 4acy + 2azy - 12aac - 4acc, \text{ and consequently } BF(y) = \frac{3aaz + 2acz - 6aac - 2acc}{9aa + 6ac - 3az + cc - cz}.$$

Now this must be equal to the focal Distance of *parallel Rays* reflected from a *concave Speculum*, which by *Prop. III.* is just half the Radius; whence we have another Value of $BF(y) = \frac{a}{2}$.

And comparing both together, we have $\frac{3aaz + 2acz - 6aac - 2acc}{9aa + 6ac - 3az + cc - cz} = \frac{a}{2}$, or $9az + 5cz - 18ac - 5cc = 9aa$, and consequently the Radius required $CD(z) = \frac{9aa + 18ac + 5cc}{9a + 5c}$. Q. E. I.

Fig. 3. If then we would *construct* this Value of $CD(z)$, we have the following Propor-

Proportion given us for that Purpose $9a + 5c$: $\sqrt{9aa + 18ac + 5cc}$:: $\sqrt{9aa + 18ac + 5cc}$: z . Wherefore making the rectangular Triangle M L K in such manner, that L M shall be equal to $9a + 5c$, and L K equal to $\sqrt{9aa + 18ac + 5cc}$; and then drawing from K the Perpendicular S K, the right Line L S is the Radius required. For by *Element. VI. 8.* $LM (9a + 5c) : LK (\sqrt{9aa + 18ac + 5cc}) :: LK (\sqrt{9aa + 18ac + 5cc}) : LS (z)$. And consequently the *Construction* gives the true Value of C D (z) as before found. Q. E. D.

Fig. 4. The Radius required is also capable of another *Construction*; for making an actual *Division* of $9aa + 18ac + 5cc$ by

$9a + 5c$, the *Quotient* is $a + c + \frac{4ac}{9a + 5c}$:

And if A B (a) be sufficiently great in respect to B D (c), the Term $5c$ in the *Denominator* of the *Fraction* may be neglected; and then it becomes $z = a + c + \frac{4}{9}c$, whose Excess above the Truth is not at all sensible. Wherefore taking D O equal to $\frac{4}{9}$ of B D (c), and making A C equal to D O, C D ($a + c + \frac{4}{9}c$) is very nearly the true Value of the *Radius required*. Q. E. D.

To shew how near this last Value of $z = a + c + \frac{4}{9}c$ is to the Truth, if we suppose the Thickness of the *Speculum* to be $\frac{1}{3}$ of the given Radius of the first Surface, which is
very

very considerable : In this Case the exact value of $z = a + c + \frac{4ac}{9a + 5c}$ is but $\frac{2}{225} a$ or nearly $\frac{1}{12} a$ less than its Value found by neglecting $5c$ in the other Equation $z = a + c + \frac{4}{9}c$, a Difference not at all considerable in *physical* Matters.

If c be supposed equal to $\frac{1}{3}a$, which is still a far greater Supposition, even in this Case, the Value of the Radius z taken from the last Equation $z = a + c + \frac{4}{9}c$, is but $\frac{15}{268} a$ or very nearly $\frac{1}{42} a$ greater than the Truth; which is an Excess not very sensible, unless a be extremely great. But if the Thickness be greater than in this last Supposition, it will be convenient to take the Value of the Radius required from the exact Equation

$$z = a + c + \frac{4ac}{9a + 5c}.$$

As, if c be equal to $\frac{1}{2}a$, the Radius required is $1\frac{3}{46}a$; which is about $\frac{1}{21}a$ less than what it would be if $5c$ were neglected. If c be equal to a , the Radius required is $2\frac{2}{7}a$; which is about $\frac{1}{6}a$ less than it would be if $5c$ were neglected. If the Thickness be so considerable as to be equal to thrice the given Radius of the first Surface, then the Radius required is $4\frac{1}{2}a$.

Hence likewise, if the Radius of the last Surface be given, together with the Thickness of the *Speculum*, we may find the Radius

dus of *Concavity* necessary to unite the two Images of a vastly distant Object made by *Reflexion* from both Surfaces. For if c be not very great, we shall have $a = z - c - \frac{1}{3}c$, as near the Truth as need be required in Practice: For if we had the exact Value of a , we could not in Practice grind the *Speculum* to the due Concavity, even so near as the Value just now given. If c be considerable, the Value of a must be found from an Equation of an higher Degree $9aa - 9az + 18ac = 5cz - 5cc$; which if it be contracted, by putting p for $2c - z$,

$$\text{will give } a = \mp \frac{1}{2}p \pm \frac{\sqrt{5cz - 5cc}}{9} + \frac{1}{4}pp,$$

the Sign of $\frac{1}{2}p$ being either $-$ or $+$, according as $2c$ is greater or less than z . After the same manner, having the Radius of both Surfaces given, we may find what Thickness of the *Speculum* is necessary to unite the two Images of a *vastly distant Object*, formed by *Reflexion* from both Surfaces, by means of the following Equation, $5cc + 18ac - 5cz = 9az - 9aa$; which if it be contracted, by putting q for $3\frac{1}{2}a - z$, will give $c = \mp \frac{1}{2}q \pm \frac{\sqrt{9az - 9aa + \frac{1}{4}qq}}{5}$.

VII. A more particular Account of MICROSCOPES and TELESCOPES, from Mr. *Huygens*.

PROP. I.

TO explain the Effects and Uses of single Microscopes, and the Manner of making little Spheres and Lenses.

[Fig. 9, 10.] Let N be the *Lens* QRQ, the Object at its *Focus* R, O the Eye very near the *Lens*. The Rays coming from R will after their Refraction fall parallel upon the Eye, and consequently make distinct Vision. For the Fabrick of the Eye, having its *focal* Distance just at the Bottom of it upon the *Retina*, requires that the Rays from each single Point should fall nearly parallel in order to be there collected; that is, that the Basis of each Cone of Rays flowing from every Point of any Object, which Basis is the Pupil of the Eye, should bear so small a Proportion to the Length of the Cone, as that those Cones may be looked upon as little Cylinders. (The Distance indeed requisite for distinct Vision is not limited to a Point, but is indulged in larger Bounds; because Nature has furnished us with the Power of contracting the Pupil as the Object

ject comes nearer, and so diminishing the Basis of each Cone in proportion, and consequently of preserving distinct Vision; but this is only to a certain, and that no very great Degree.) But the Object QRQ will appear in the same Magnitude as if the *Lens* N were removed, and a Plate with a small Hole in it substituted in its Place, namely under the Angle QAQ . So that all that the interposed *Lens* does in this Case, is only making distinct Vision, which would without the *Lens* be confused. But since at the Distance suppose of 8 Inches from the Object, a naked Eye has then distinct Vision; the apparent Image may be said to be so much magnified as those 8 Inches exceed the little Space NR , or the *focal* Distance of the *Lens* N : Which if it be equal to $\frac{1}{5}$ of an Inch, the Appearance of the Image seen distinctly by the *Microscope* is to that seen distinctly by the naked Eye, as 40 to 1. Therefore the less the *focal* Distance of the little *Lens* N is, the greater will its Effects be in dilating the Image of a small Object; though there are some Inconveniences (to be mentioned afterwards) which here offer themselves, and forbid our going beyond some certain Limits. And the same thing happens to little *Spheres*, which may be used for *Lenses*, and might otherwise be made as little as we please. But these small *Spheres* are inferior to little *Lenses* upon this Ac-

Y 2

count,

count, that for the same Degree of magnifying, if both be made of Glass, the *Lenses* are three times more distant from the Object than the *Spheres*; and by that means leave a sufficient Space for the lateral Light to enter, and make the Colours of the Object visible; whereas otherwise we are forced to turn the *Microscope* directly against the Light, and can only then discern distinctly such Objects as by their Thinness are pellucid.

The Effects of a little *Sphere*, and what has been said concerning the three times less Distance, is thus demonstrated. *Fig. 11. 9.*

Let there be a Glass *Sphere* whose Center is K, and its Axis A B, in which produced on both Sides the Eye is placed at D, and the Object at C, each of the Distances A D, B C being taken equal to half the Radius A K: And consequently the Point C is the *Focus* where Rays falling parallel to the Axis A B, upon the *Sphere* at A H are after Emerision collected; as is shewn in Article IV. of this *Supplement*. Wherefore an Object placed at C will send Rays upon the *Sphere*, which will after Refraction be received parallel by the Eye, and consequently make distinct Vision. But by *Prop. XIV.* if we take the Point F such, that F A may be equal to the Radius A K, the Point F is the *Focus* towards which parallel Rays after Refraction at the first Surface B G do tend
in

in their Passage through the *Sphere*, and from which they are diverted after Refraction at their Emerfion, and collected at D. Make G E parallel to the Axis, and comprehending the Portion of the Object C E, and draw the right Line E D. The Ray then E G being refracted at G, proceeds according to G F; and being again refracted at H, goes on to meet the Eye at D. Wherefore the Line C E is feen under the Angle A D H, which would appear to the naked Eye under the Angle C D E; which I fay is but the half of the former.

For becaufe A F is double of A D, the Angle A D H is double of A F H. But D F is parallel to E G, becaufe G E is both parallel to F D, and to be looked upon as equal to it, or to the right Line B C; becaufe C E is to be a Line very fmall with refpect to the Diameter of the *Sphere*. Therefore the Angle A D H is alfo double of the Angle C D E, and confequently equal to the Angle C K E. From whence it is plain, that to the Eye placed at D the Line C E will appear under the fame Angle in which it would appear to the naked Eye feeing from the Point K. Whence if the Diameter of the little *Sphere* A B were $\frac{1}{12}$ of an Inch, we fhould have K C = $\frac{1}{6}$ of an Inch; which is to the Distance of 8 Inches in the Proportion of 1 to 128: So that the increafed Magnitude of the Object would be as 128 to 1; which is indeed
very

very considerable. But if NR , the *focal* Distance of the *Lens*, be equal to the right Line KC , we have shewn that by this means the Object RQ would be seen in the same Magnitude as if the Eye were placed at N without the *Lens*; nor in the using this *Lens* will the apparent Magnitude be any ways changed, in whatsoever Part of the Axis RN produced the Eye be placed. Therefore 'tis plain the same Degree of Magnifying, and the same Effect every way, is performed equally by the *Lens* N and the little *Sphere* AB . And it is moreover manifest, that the Distance RN , being taken equal to KC , is equal to thrice BC . *Q. E. D.*

We are next to explain the Manner how little Spheres and Lenses may be prepared and fitted for Use.

The less *Spheres* are to be, the easier they are prepared, after the following manner. Take the smallest Fragments of Glass, and hold them to the lower part of the Flame of a Candle, where the bluish Colour is discernible, that they may grow red hot; and then if they be taken up by the finest Steel-Wire that can be got, and dexterously turned, they will be formed into Globules, which are large enough if equal to a Grain of Mustard-Seed. Out of several thus prepared, you will find some very good; which may be tried by including them in a Brass-Plate, and is thus done:
Take

Take a Plate of the thinnest Brass the Breadth of a Finger, and twice as much in Length, and bend it double; perforate this Rectangle in the middle with the Point of a Needle, and rub the opposite Holes smooth with a Whetstone, that no Roughness may remain about the Edges, and black them with the Smoak of a Candle, that no Brightness may continue within. Put the little *Sphere*, still adhering to the Steel-Wire, into the Holes within the Brass-Plate, and fix it there, by fastening the two Leaves of the Plate together. After this manner you may make several *Microscopes* with great Ease, out of which you may make choice of those that are best.

The principal Use of this sort of *Microscopes* is, to look at Corpuscles that are pellucid. And they are placed in a *Machine* made in such manner, that by turning a Screw they may approach to or recede from the Object, and so be brought to the due Distance, which is requisite for distinct Vision. And to this it conduces very much, that the too great Light be restrained, and only admitted through the Hole, which is about four times the Distance from the Object. For by this means the Aperture of the *Lens* is better limited than by the Breadth of the contiguous Hole, which there is no Necessity at all of straitening. The Eye must be brought as close to the
little

little *Sphere* as may be; that it may comprehend the greater Space.

The Corpuscles or Drops of Liquors which are to be looked at, are put upon a little circular Plane of Glass, which is made to slide laterally every way, that we may bring every Part of the Object to be viewed successively. Some attract the Liquor to be examined into *capillary* Tubes of Glass, so small as scarce to admit an Hair, which has likewise its Uses. But in using those little *Lenses* before mentioned, Care must be taken, that while by means of another *Lens* on one Side we cast Light upon the Object, the Hole of the Aperture may be exactly limited, by trying how much it may lie open without being an Hinderance to distinct Vision. For here the Points of Corpuscles emit Rays of Light, and are so many *radiant* Points, which is quite otherwise in those pellucid Corpuscles that are looked at through little *Spheres*, where the Objects intercept the Light, not emit it.

The Effects of this sort of little *Lenses* and *Spheres* are very wonderful, as may be seen from those Experiments with them which have been made publick, and from which our Knowledge of Nature has received very great Light and Information. By these the circulatory Motion of the Blood has been put beyond Controversy, which our *Leewenboek*, the most dilligent Observer
of

of these Matters, has shewn me in the Tail of an Eel, to my very great Satisfaction and Delight. For the Blood appears pellucid, and consisting of reddish Globules, and runs through the Channels of the Arteries, which are continued to the Veins with a very rapid Motion. Which without doubt might be observed in all other Animals, if we could find out such Parts in them as are pervious to the Light. He put the live Eel into a Glass Tube half full of Water, to which he externally applied the *Microscope* at that Part where the Extremity of the Tail touched the Tube.

'Tis also very pleasant to observe the *Animalcula* that swim in Drops of Water, in which we have infused Ginger, Pepper, or something else of an hot powerful Odour for some Days: They are of various Forms, and some less than others; their Motions are wonderful, and for their Size sufficiently quick; nor is the Instrument apparent by which they perform them, for they have neither Legs nor Arms, nor do they bend their Bodies like Fishes. For the little Eels in Vinegar, which are much larger than they, swim like those in the River, in which it is very much to be wondered that they should generate little ones of themselves. For I saw one which had four young ones within it (for they are altogether pellucid) and after it had been kept in the Tube for

some Hours, brought them all forth, every one of which did afterwards swim by it self.

It is very probable that those *Animalcula* which I have said move about in Water, are invited thither out of the Air by the Odour of the Infusion. For the same Figures appear upon macerating several things in Water; but if the Vessel be covered, none at all appear. Nor is it difficult to conceive how they should be supported in the Air, when they are so much smaller than the finest Dust that is. So that perhaps we draw many thousands of them into our Lungs every time we fetch our Breath, without knowing it. Nor would it be useless to observe at what time of the Year they appear in greatest Numbers, and whether they encrease in a vitiated Air. Milk appears to consist of small pellucid Globules swimming in a Liquor likewise pellucid, but of a different Refraction; and hence it is that it appears white, though it contains no other Matter but what is perfectly transparent, and without Colour.

I omit those many wonderful Forms of minute Insects; the Wings of Butterflies and Gnats, covered with little Feathers; the Powders observed in the Middle of Flower-Tops, which are nothing else but little transparent Bladders, filled with that Matter of which the Bees make their Hony, and

and which they carry between their Legs into their Hives. But what ought to be looked upon as the most wonderful and astonishing of all is, that an immense Multitude of *Animalcula* are discovered to swim in the *Semen masculinum*, after the manner of little Fishes, almost of the same Figure with a Frog newly formed, and yet without Legs. Which *Animalcula*, I make no question, enter the *Ova muliebria*, and are the Rudiments of what is brought forth from thence. There are several Considerations which confirm this Opinion; nor is it any great Objection, that out of so great a Multitude either few or only one of them comes to Maturity, and grows to be an Animal; since the same abundance and superfluous Fruitfulness is equally observable in most Seeds of Trees and Herbs, as of Firr, Poppies, &c.

These *Animalcula*, by reason of their wonderful Smallness (for even ten thousand of them are not equal to the smallest Grain of Sand) ought to be looked at through such Glass Globules as have the greatest Power of Magnifying.

PROP. II.

*To explain the Effects of Compound
MICROSCOPES.*

WE come now to speak of *compound Microscopes*, by the Help of which such Objects as are not transparent are looked at, and their true Colours discovered, and that much better and more commodiously than through single *Lenses*.

[*Fig. 12. 13.*] Suppose the *Microscope* be a double one, consisting of two *Lenses*, one less A, and another greater B. Why we dispose them so, we shall afterwards explain. And let B be the ocular *Lens* nearest the Eye, placed suppose at C; A the Object *Lens* nearest the Object placed suppose at E; and A B C the common Axis of both *Lenses*. There will be two Cases, as may be seen represented in the two Figures to which this *Prop.* refers. In the first, Rays proceeding from a single Point E of the Object, and falling upon the *Lens* A, are refracted by it and again united in the Point P, and there intersecting one another, and proceeding towards the *Lens* B, are by it refracted and made parallel, and so enter the Eye at C, and by that means make distinct Vision. 'Tis necessary therefore that A E, the Distance
of

of the Object, should be greater than AQ , the *focal* Distance of the *Lens* A . And the *Focus* P must be found by *Prop.* XVI. or by making EQ, EA, EP , in a continual Proportion. But the *Lens* B is to be so placed, that its *Focus* on that Side towards A may fall exactly upon the Point P , in order that the Rays may be made parallel after Refraction at the *Lens* B . All which is easily done by what has been before demonstrated. The other Figure (13) represents the several Rays DAG, FAH, EAB , proceeding from different Points of the Object. A is the middle Point of the *Lens*, and AP, AB, AC , are made in a continual Proportion in order to determine the Place of the Eye C ; for by this means, however small the Aperture of the little *Lens* A may be, the whole *Lens* B will nevertheless be filled with the Image of the Object, because the Rays falling from A upon the whole *Lens* B are collected in the Point C .

[*Fig.* 13.] But the Proportion of the apparent Magnitude to the true will be found by drawing the right Line CF . For the Proportion required will be the same to that which the Angle BCH bears to the Angle ECF ; which Proportion is compounded of the Proportion of the Angle BCH to BAH , and of the Angle BAH or EAF to the Angle ECF . But the first of these is the same with the Proportion of the right Line AB to BC , and the last that of CE to EA ;

EA; because Angles that are small are looked upon to be to one another as their Tangents. Therefore the Proportion of the apparent Magnitude to the true will be compounded of the Proportions AB to BC, or AP to PB (for AP, AB, AC, are in a continual Proportion) and CE to EA. But that the Effects of the *Microscope* may be more exactly estimated, the Angle BCH is rather to be compared with the Angle under which the right Line EF would be seen at the Distance of 8 Inches from the Eye; that is, with the Angle ELF, LE being taken equal to 8 Inches, according to what has been said before of magnifying by a single *Lens*. And consequently the Proportion of magnifying must be here understood to be compounded of the Proportion of the Angle BCH to BAH, and BAH or EAF to ELF; that is, of the Proportion of AP to PB, and of EL, a Line 8 Inches long, to the right Line EA. For if the *Microscope* were of so great Length, that, for Example, CE should be two Foot long, that is, equal to thrice the right Line LE, and the apparent Magnitude to the true were found by the former Reasoning to be as 90 to 1; yet it is not really any greater than as 30 to 1, because the right Line EF would only appear 30 times greater by the Assistance of the *Microscope*, than it would if viewed by the naked Eye at the Distance of 8 Inches:

For

For we are not to consider how much, by means of the *Microscope*, we magnify an Object at the Distance of two Foot; but how much greater it is made than when viewed at that Distance to which we bring our Eye when we desire to look at any thing more curiously.

*Of the LIGHT and APERTURE of
MICROSCOPES.*

UPON the *Aperture* of *Microscopes* all their Effects and Virtue entirely depend: So that from hence it is that we are to learn to what Degree the magnifying of Objects may be brought; which no body that I know of has hitherto determined. And it will be found that we may here proceed *ad infinitum*, as shall be shewn in *Telescopes*, not indeed in a single *Microscope* of one little *Lens*, but in those which are made by a Combination of more than one.

In *Microscopes* made of a single *Lens* it is to be observed, that if their *focal* Distance be about half an Inch or greater, there will be no occasion for limiting the *Aperture* in order to make distinct Vision; because the very Narrowness of the Pupil of the Eye excludes, as much as there is occasion, those Rays that disturb Vision, and as much as they would be excluded if the *Lens* were

were made to have a less *Aperture*. But in smaller *Lenses*, where this Limitation of the *Aperture* is necessary, the Rule is, that the Diameters of those *Apertures* should be in the same proportion with the *focal* Distances of the respective *Lenses*, in order to have the Object seen by both equally distinct. But the *Light* or *Brightness* will be in a duplicate Proportion of those *focal* Distances; so that the more convex the *Lens* is, the greater indeed, but then the more obscurely will every thing be seen.

[*Fig. 14.*] Let *P* be a small *Lens*, whose Axis is *T B F*, *P D* the Semidiameter of the Aperture, which Experience teaches to be the greatest that can be admitted, and that less than the Pupil of the Eye, *F* the extreme *Focus* of red Rays (which are least refrangible) proceeding parallel to the Axis, in which Point suppose the Object to be placed, and *B* the *Focus* of violet-coloured Rays, which are most refrangible. The same things being supposed in a smaller *Lens p*, the Semidiameter of whose *Aperture p d* is to the *focal* Distance *p f* in the same Proportion as in the greater; I say the Object will be seen equally distinct in both.

For since the Ray *E D*, parallel to the Axis falling upon the *Lens P*, is refracted unequally, and divided into its extreme Colours by the Angle *F D B*, so that the extreme Colour Red passes to *F*, and the extreme

extreme Violet-colour to B; it will happen on the contrary, that a Ray F D proceeding from the Object will be divided into its extreme Colours by an Angle E D K, equal to F D B. Therefore in both Cases F D B is the *Angle of Aberration*, upon which depends the Aberration of the Rays in the bottom of the Eye, as shall be shewn when we come to speak of *Telescopes*. But since from the Nature of this *Aberration*, P F is to F B as pf to fb ; and also by Construction P D is to P F as pd to pf ; it follows that the Angles, as well P F D, pdf , as P B D, pbd , are equal. Wherefore the Difference of the Antecedents, P F D, P B D, is equal to the Difference of the Consequences, pdf , pbd ; that is, the Angle F D B is equal to the Angle fdb , and consequently the *Aberrations* in the bottom of the Eye are in both Cases equal, and by that means Vision equally distinct.

Moreover, because the Angles P F D, pdf are equal, it is plain that the same Quantity of Rays in both Cases proceeds from the same Points of the Object F and f , or any others, upon the *Lenses*, and from thence to the Eye. But the Breadth of the Object in the bottom of the Eye is in the smaller *Lens* so much greater as P F is greater than pf , as has been before demonstrated; and the apparent Surfaces are in a duplicate Proportion of those Breadths.

Therefore the same Quantity of lucid Rays expended towards illustrating each Surface, will make that which is least the clearest by so much as the other Surface is greater; that is, in a duplicate Proportion of $P F$ to $p f$; which was the last thing to be demonstrated.

Since therefore the same Perfection of Vision which is to be found in larger *Lenses* cannot be had in more convex *Lenses* without diminishing the Brightness of the Object at the same time; it follows that we cannot proceed in magnifying as much as we please, unless a greater Light be borrowed somewhere else to illustrate the Object. Nor will this be of any great Benefit, because the Latitude at the Pupil of the Eye, or the little Cylinder of Rays flowing from every Point of the Object, and which has here the same Latitude with the Aperture, cannot be contracted farther than the fifth or sixth part of a *Line*; so that even this limits the Efficacy of these little *Lenses*.

The Effects of more compounded *Microscopes* will easily be accounted for after the same manner. And indeed a full Consideration of *Prop.* XXI. and XXII. is sufficient for explaining the Effects of all sorts of compound *Microscopes*.

Of TELESCOPES.

PROP. III.

A Telescope made by a convex and concave Lens represents vastly distant Objects distinct and erect; and magnifies them according to the Proportion of the focal Distance of the convex Lens to the focal Distance of the concave Lens.

Fig. 15. Let AO be the common Axis of both Lenses, and A the extreme convex Lens, whose Focus of parallel Rays proceeding from the vastly distant Object is supposed to be at O. Let D be the concave Lens, which is so placed between the Lens A and its Focus O, that the same Point O may also be the Focus of the concave Lens, where Rays falling parallel from the Side of O would be collected. And first, suppose the Eye of the Spectator placed next to this Lens.

The Rays then proceeding parallel from each Point of the vastly distant Object, and falling upon the Lens, those which proceed from that Point of the Object, which is in the Axis produced, would be collected at the Point O; but they are again made parallel by means of the Lens D. We would have

the Rays fall parallel upon the Eye, that the Telescope may be fitted for those who have good Eyes; for we shall speak afterwards of short-sighted Eyes. In like manner the Rays proceeding from those Points of the *vastly distant Object* which are out of the Axis would be all collected at respective Points near O; but these also by *Refraction* at the *Lens* D are again made parallel, though something oblique to the Axis A D, which Rays, to avoid Confusion, are not expressed in the Figure. Therefore the Rays which proceed from the *vastly distant Object* being made to fall parallel upon the Eye, will make *distinct Vision*; and since those Rays that proceed from the *Object* go on to meet the Eye in the same Order, 'tis plain the apparent Proposition of the *Object* will be the same with the true, or the *Object* will be *erect*.

Fig. 16. The *Lenses* A C and D, and the Point O being placed as before, find by *Prop. XVI.* the Point P, to which Rays tending, will, by *Rafraction* at the *Lens* A C, be collected at D the Center of the *Lens* D: Which Point is also found, by making D P a third proportional to D O, D A, and taking it on the same Side with D O. Suppose the Ray E C P to be one of those which proceed from the extreme right Side of the *vastly distant Object*, which
 imagine

imagine to be the Moon, and its Center to be placed in the Axis DA produced. It is plain that this Ray will come to the Eye in the right Line $CD F$, because it passes thro' the Center of the *Lens* D , whose middle Thickness may be neglected as inconsiderable, and its two Surfaces in that Place looked upon as parallel. But we have shewn before, that all the Rays proceeding from each Point of the Moon will, by means of such a Telescope, fall parallel upon the Eye. Wherefore the Eye will receive all the Rays from that Point E of the Moon in such manner, as that they shall be parallel to the Ray $CD F$; and consequently will see that Point of the Moon in the Place to which the right Line DC tends; which tending to the same Side of the Axis, on which that Point of the Moon is situated, from whence the Rays proceeded, 'tis plain the Object will appear *erect*. Moreover the Angle ADC determines the Semidiameter of the Moon, as encreased by the Telescope. But the Angle CPA is that which determines its Semidiameter, as seen by the naked Eye; because we before supposed the Ray $EC P$ to proceed from the extreme right Side of the Moon, and the Ray $HA P$ from its Center: For though the Point P is beyond the Eye, and the Eye sees from the Point O , yet the Moon being an Object vastly distant

stant, will appear under the same Angle to the naked Eye, whether it be viewed from the Point P or O. Therefore the Moon will appear magnified, according to the Proportion of the Angle $A D C$ to $A P C$, which Proportion may here be looked upon as the same with that of $P A$ to $D A$. But because by Construction $D O$ is to $D A$ as $D A$ to $D P$, by inverting and compounding the Proportion, $A O$ will be to $O D$ as $P A$ to $A D$. Wherefore the apparent Magnitude will be to the true as $A O$ to $O D$. *Q. E. D.*

It appears from hence, that the apparent Magnitude is the same, in whatsoever Place behind the *Lens* D the Eye is situated.

Fig. 17: Let the *Lenses* A C and D be placed as before, and let A Q be taken in their Axis produced equal to A O. And out of those Rays which proceed from a Point of the right Side of the Moon, let us consider the Ray R Q C passing through the Point Q (for some one will pass through it) and meeting with the *Lens* A C in C. It will afterwards become parallel to the Axis A D, and when refracted again at the concave *Lens*, will diverge, as if it came from the Point L, and will tend to the Eye in the right Line L I F, so as that the Distance L D may be equal to D O; because L is in that Case the *Focus* of parallel Rays falling upon the *Lens* D.

The

The Proportion of the encreased Magnitude is hence easily collected. For because the Rays proceeding from the right Extreme of the Moon, after having passed both *Lenses*, are parallel, and come so to the Pupil GF , and consequently they become parallel to the Ray LIF , which we know to be one of them; that Point of the Moon will appear in the right Line IL , and consequently the Semidiameter of the Moon will be comprehended in the Angle ILD . But the Angle in which the Semidiameter would appear to the naked Eye, either from D or from Q , is RQH , or CQA . Therefore the Proportion of the encreased Magnitude is the same with that of the Angle DLI to AQC ; that is (because of AC , DI equal) as AQ to LD . But AQ is equal to AO , and LD is equal to DO . Therefore the Proportion of the apparent Magnitude to the true is as AO to OD . *Q. E. D.*

To determine what will be the Amplitude of the visual Angle, or of the Space which is represented at one View by a Telescope consisting of a convex and concave Lens.

Fig. 18. The Amplitude of the *visual Angle* in these *Telescopes* depends chiefly upon the Magnitude of the Pupil of the Eye, which is confirmed by Experiment. For if applying your Eye to the Telescope,
you

you first shut it, that the Pupil may be dilated as it usually is in the Dark, and then open it on a sudden, at first View you will discern Objects in a larger Orb than a little while afterwards, the Orb being presently contracted as soon as the Pupil is contracted by the Brightness of the Light. But if you place a Plate perforated with a small Hole before the Eye, you will discern every Object in a lesser Orb.

If you make the Hole extremely small, the lucid Orb will not be contracted in proportion to the Smallness of the Hole, but its Amplitude will then be limited by the Aperture of the convex *Lens*, and consequently will not be diminished beyond a certain Degree, except the convex *Lens* be also more contracted. The Reason of which is very easy to be explained. For if EF be the convex *Lens*, and B the concave; to which the Pupil of the Eye applied has first the Magnitude CD ; draw from the opposite Points C, D in the Circumference of the Pupil, through the Center of the *Lens* A , the right Lines CAH, DAG . These will determine the *visual Angle*, under which that Part of any Object which is seen at one View is comprehended: Because Rays coming from the Points G, H through the Center of the *Lens* A , penetrate without Inflexion to C and D ; therefore

fore that Part of the Object which is comprehended within the Angle $G A H$ cannot but send Rays to the Eye, even though the Pupil were a little narrower than $D B C$. For drawing $G A K$ so as to make $A K$ equal to $A O$, and joining $E K$; if so be $E K$ falls upon the Pupil, the Object comprehended under the Angle $G A H$ will be discerned, but the extreme Points towards which the right Lines $A G$, $A H$ tend will be seen but obscurely, because only a small Part of the Rays, which they cast upon the *Lens* $E F$, enter the Pupil. And hence it happens, that how much soever the Aperture of the *Lens* $E F$ is contracted, the Amplitude of the *visual Angle* is nevertheless not at all, or extremely little diminished, so the Orb of the Pupil be not contracted. But this Breadth of the Pupil being diminished, and reduced as it were to a Point, the Amplitude of the *visual Angle* is the same with that of the Angle $E P F$; $E F$ being supposed the Aperture of the convex *Lens*, and the Point P found by *Prop.* XVI. or by making $B O$ (the Distance of the concave *Lens* from the *Focus* of the convex) $B A$ and $B P$ in a continual Proportion. For no Rays transmitted through the *Lens* A can arrive at the Point of the Eye B , but such as before they fall upon that *Lens* tend towards the Point

$B b$
 $P.$

P. The greatest Angle EPF, of which Rays, is determined by the Aperture of the *Lens* A.

This is the *Telescope* which was first found out by *Galileus*, and still retains his Name; and is the same with a common Prospective Glass.

PROP. IV.

A Telescope made of two convex Lenses represents vastly distant Objects distinct, but inverted; and magnifies them according to the Proportion of the focal Distance of the exterior or object Lens, to the focal Distance of the interior or ocular Lens.

[Fig. 19, 20.] Let AC be the exterior convex Lens, D the interior, AD the common Axis of both, and O the Focus of the Lens AC. Let the other Convex D be so placed, that the same Point O may be also its Focus, or the Point of Concourse of parallel Rays coming from the Side of G, where the Eye is supposed. We are to shew that all this being supposed, vastly distant Objects will be seen distinct, and inverted and magni-

magnified, according to the Proportion of AO to OD .

And here we must make use of two several Figures, as in the *preceding Proposition*; in the first of which the Rays coming parallel to the Axis HA , are by the Refraction of the Lens AC collected at its *Focus* O , and from thence tending farther to the *Lens* D , are by it again made parallel to the Axis AD , and so come to the Eye placed at G . And as in the *preceding Prop.* we must again consider this Composition of parallel Rays, as coming from a single Point of the vastly distant Object, which is placed in the Axis HAD , as suppose from the Center of the Moon; and the like parallel Rays coming from every other Point of the *Object* upon the *Lens* AC , as suppose from the extreme right Side of the Moon, which are inclined to the former, and being thereby refracted, are collected in a Point of the Axis near O ; where intersecting themselves, and proceeding to the *Lens* D , after they have gone through it, they are again made parallel, (that is, only among themselves respectively) and so arrive at the Eye. Whence 'tis plain, Vision will be made distinct.

The other Figure shews the inverted Situation, and the Proportion of the encreased Magnitude of the *Object*. Where the con-

vex *Lens* A C and D, and their common *Focus* O, being placed as before; and moreover, as in the second Demonstration of the *preceding Prop.* the Distance A Q being made equal to A O, the remaining Part of the Demonstration will proceed much after the same manner. For if, out of the Rays which proceed from a Point in the extreme right Side of the Moon, we choose one R Q C passing through the Point Q; that, after Refraction at the *Lens* A C, will pass in C I parallel to A D, and being again refracted by the *Lens* D, will tend along the right Line I F L to the Point L, taken in such manner that the Distance E L is equal to D O. But because the Rays from the extreme right Side of the Moon, after Refraction at both *Lenses*, arrive parallel at the Eye, as has been said before, and I F L is one of them; it follows that they will all fall parallel to I F L upon the Eye, and that Point of the Moon will be seen in a Place, according to the right Line F I: Which since it tends to the opposite Side to that from whence those Rays came, 'tis plain that the Situation of the Moon will appear inverted, so that the right Side will be changed to the left, and the upper Parts to the lower. Moreover, since the Center of the Moon will be seen in the right Line D A, I L D will be the apparent Angle of the Semidiameter

meter of the Moon. But to the naked Eye that Semidiameter is comprehended under the Angle HQR , or AQC . Therefore the *Ratio* of the apparent Magnitude to the true is as the Angle DLI to AQC ; that is, as AQ to DL ; because AC , DI are equal, that is, as AO to OD . *Q. E. D.*

And here likewise it appears, that it signifies nothing to the apparent Magnitude, wheresoever the Eye is placed behind the *Lens* D . But that it may comprehend most at one View, it is convenient it should be placed at or near the Point L ; because it appears, that although the Breadth of the Pupil be ever so little, yet the whole *Lens* D , while it does not exceed the Aperture of the *Lens* AC (for it is usually confined within this Measure) will be seen full of the Object.

This is the *Telescope* most commonly used to look at celestial Bodies.

PROP. V.

TO explain the Construction of a Telescope compounded of four Convexes, by means of which Objects are seen erect and very amply.

[Fig.]

[Fig. 21, 22.] *Telescopes* made of *two Convexes*, because of their inverging the Position of the Object, are seldom used, except in observing the Stars, the Position of which is not regarded. The Proportion in which this Sort magnifies the Object has already been demonstrated. But if we would have these Images again made *erect*, and at the same time a great Share of them be represented to the Eye at one View very *amply*, we must use 3, 4, 5, or more *Lenses*. Which however are not to be multiplied without Cause, because the Matter of each of them and the Reflexion of their several Surfaces divert Part of the Rays. But we cannot obtain the desired Effect perfectly, with fewer than 4 *Lenses*. For although in the same Length of the *Telescope* both an *erect* Situation and the same Degree of Magnifying, and an equal Share of the Object may be had as well with 3 as 4 *Lenses*; yet the Composition of 3 *Lenses* is much more inconvenient than that of 4, because in that the two ocular *Lenses*, or at least that which is next the Eye, must be made of larger Segments of a Sphere, with respect to its Diameter, or to the *focal* Distance, if the same Magnitude of the visual Angle be required. And hence the Objects come to be coloured, and right Lines, at the Margins of the Aperture, appear curve.

There-

Therefore we must make our Telescope of 4 *Lenses*; which is done after the following manner.

The exterior or object *Lens* is A, whose *focal* Distance is A B; and in the same Axis are placed three ocular *Lenses*, C, D, and E, all equal to one another, the inmost of which is placed beyond the *Focus* B, by its *focal* Distance B C, and the next D is placed beyond C, by twice that Distance, B C, and the last as far from D as that was from C; and lastly, the Eye must be placed beyond this last by the Distance B C.

There is here again Occasion for two Figures; in the first of which are represented Rays proceeding from a single Point of the vastly distant Object: Which ('tis plain to any who understand what has gone before) first fall as it were parallel upon the *Lens* A, and are by it collected at its *Focus* B, and thence diverging fall upon the *Lens* C, which makes them again parallel, and throws them upon the *Lens* D, which collects them at its *Focus* H, the middle Point of the Distance D E; from whence proceeding on to the *Lens* E, they are by it made a third time parallel, and being received so by the Eye F, they make distinct Vision, by being collected at its *Focus* which is in the bottom of the Eye.

The other Figure considers the *Proportion* of *Magnifying*; which is, *That which*
A B,

AB, the focal Distance of the object Lens, bears to BC, the focal Distance of one of the ocular Lenses: And demonstrates likewise the Amplitude of the visual Angle. For the Apertures of the three ocular Lenses being supposed equal, which must not exceed the Aperture of the object Lens A, draw MQ, NR parallel to the common Axis, and comprehending the Diameters of the Apertures of the Lenses E and D. And also KO, LP parallel to the same Axis, and comprehending KL, the Aperture of the Lens C; and taking AG equal to AB, draw the right Lines OGU, PG T intersecting one another in G. Now it is evident the Latitude of the Object, which if seen by the naked Eye from the Point G, and consequently from F also, the Distance of the Object being as it were infinite, would appear comprehended in the Angle TGV; if seen through the Telescope, would appear comprehended in the Angle MFN: And consequently the Proportion of the apparent Magnitude to the true is as the Angle MFN to the Angle TGV, or PGO; that is, PO and MN being equal, as the Distance AG to the Distance EF; that is, as AB, the focal Distance of the object Lens, to BC, the focal Distance of one of the ocular Lenses. Q. E. D.

It appears moreover, that the visual Angle MFN comprehends the same Latitude
of

of the Object with a Telescope made of two *Lenses* only, A and C; for that Share of the Object which is comprehended in the Angle T G V, would be seen through that Telescope in the Angle K S L equal to the Angle M F N.

This incomparable Composition of *Lenses* was found out by I know not whom at *Rome*, and may be much improved by placing an *Annulus* or Ring either at H, the common *Focus* of the *Lenses* D and E, or at B the common *Focus* of the *Lenses* A and C; which is especially of very great Use in measuring the Diameters of Planets. For this *Annulus* does therefore exactly circumscribe the Circle of the apparent Images, because it cuts off those irregular Rays which are not collected near enough to B or H, and consequently are not by means of the succeeding *Lenses* sent parallel to the Eye, which distinct Vision requires: And the Colours likewise near the Margins are by this Contrivance taken away, which without it are not well to be avoided.

It may seem a little strange, that the Colours of the Iris arise no more in this Telescope, by the Refraction of so many ocular Lenses, than in that where there is but one; but to any one that shall consider it, the Reason will be very obvious. For the Lens Q R corrects and takes away those Colours,

C c

which

which the Lens *KL* produced, their spherical Surfaces being equal by Construction.

Of the APERTURE of the LENSES.

Since the Proportion of magnifying in *Telescopes* made of two Lenses has been shewn to be that which the *focal* Distance of the object *Lens* bears to the *focal* Distance of the ocular *Lens*, it may be thought perhaps, that however short the *Telescope* be, the Object may be magnified in any assigned Proportion. But there are two Causes which make this impossible; one is, that the *Aperture* of the object *Lens* remaining the same, the more we magnify the Object by using a less convex ocular *Lens*, the more obscure we make them appear. The other is, that it represents them less distinct. And if we expect a Remedy by encreasing the *Aperture*, the Confusion will be the more encreased. What belongs to the *Brightness* or *Obscurity*, will be easily understood by attentively considering the Image of any Object painted upon the bottom of the Eye; which the greater it is made, whether by means of the Refraction of *Lenses*, or only by approaching nearer, in so much greater Plenty must the Rays from every Point be received within the Eye, in order that the same *Brightness* may still remain.

main. For if looking at an Object with the naked Eye, you approach to it twice as near, the Image at the bottom of the Eye will be twice greater in Diameter, and four times in Area. But four times more Rays do also, from every Point of it, enter the Pupil of the Eye; because the Angle made by the Cone of Rays becomes twice as large: And therefore it is that the same *Brightness* of the Image is perceived at both Distances, which is the Contrivance of Nature. But if a *Telescope* were to be made which should magnify the Diameter of any Object ten times, and represent it as bright as when it is looked at with the naked Eye, the Diameter of the Aperture of the Object Lens ought to be ten times greater than the Diameter of the Pupil, although no Part of the Rays were intercepted by the Reflexion of the Surfaces of each *Lens*, or by the Colour of the Glafs. For by this means, when the Surface of the Object is magnified an hundred times, we have also an hundred times more Light than was received by the naked Pupil.

But a much less Measure of *Brightness* suffices for *Telescopes*; for those which we use in the Day-time are not too obscure, if they have but $\frac{1}{6}$ or $\frac{1}{7}$ of that *Brightness* which is usually perceived by the naked Eye. But those longer ones with which we observe

the Moon and the Planets, require not above half this last *Brightness*, because the Eye is moved with a much less Brightness in the Night than in the Day. So that in a *Telescope* 30 Feet long, which magnifies the Diameters of the Planets 109 times, and would consequently require the Diameter of the Aperture of the object *Lens* 109 times greater than the Diameter of the Pupil, that is, of about 11 Inches, if we suppose the Diameter of the Pupil to be $\frac{1}{10}$ of an Inch; we find that an Aperture of 3 Inches in Diameter suffices, which admits less than $\frac{1}{13}$ of that *Brightness* which would be admitted by an Aperture of 11 Inches.

The Proportions between the *focal* Distances of the object *Lens* (which is likewise the Length of the Telescope) the *Aperture* of the same object *Lens*, the *focal* Distance of the ocular *Lens*, and the apparent magnified Diameter of the Object, for *Telescopes* from the Length of 1 Rhinland Foot to 100, are expressed in the Table following.

T A B L E

TABLE for TELESCOPES. 203

The focal Distance of the object Lens, or the Length of the Telescope.	The Diameter of the Aperture of the object Lens.	The focal Distance of the ocular Lens.	The Proportion of Magnifying, considered as to Diameter.
Rhinland Feet.	Inches, & Decimals.	Inches, & Decimals	
1.	0,55.	0,61.	20.
2.	0,77.	0,85.	28.
3.	0,95.	1,05.	34.
4.	1,09.	1,20.	40.
5.	1,23.	1,35.	44.
6.	1,34.	1,47.	49.
7.	1,45.	1,60.	53.
8.	1,55.	1,71.	56.
9.	1,64.	1,80.	60.
10.	1,73.	1,90.	63.
13.	1,97.	2,17.	72.
15.	2,12.	2,33.	77.
20.	2,45.	2,70.	89.
25.	2,74.	3,01.	100.
30.	3,00.	3,30.	109.
35.	3,24.	3,56.	118.
40.	3,46.	3,81.	126.
45.	3,67.	4,04.	133.
50.	3,87.	4,26.	141.
55.	4,06.	4,47.	148.
60.	4,24.	4,66.	154.
65.	4,42.	4,86.	161.
70.	4,58.	5,04.	166.
75.	4,74.	5,21.	172.
80.	4,90.	5,39.	178.
85.	5,05.	5,56.	183.
90.	5,20.	5,72.	189.
95.	5,34.	5,87.	194.
100.	5,48.	6,03.	199.

PROP. VI.

TO explain the Manner of fitting a Telescope for observing Eclipses of the Sun, and discovering the Spots in its Surface, and to determine how great its Image will be represented.

[Fig. 23, 25.] A Telescope is found to be of great Use in observing *Eclipses* of the Sun, and also in discovering the Spots which are said to be in its Surface; by receiving the Image formed by both its *Lenses* upon a white Plane, from which the Light is every other way excluded. In order to explain which Invention we must first demonstrate the Position of the *Lenses* which is necessary to form the Image of the Sun, as clear and distinct as may be.

Let A B be the convex *Lens* next the Sun, whose *Focus* is E. The other is D, either concave or convex; for either of these Sorts of *Telescopes* will do the Business: Though a Telescope of two *Convexes* is the most convenient, because we make it represent the Images erect, while by the other Sort we invert them. Let the Point K be the *Focus* of the *Lens* D, where the Rays coming from the Side of H parallel, are after Refraction by it collected, and in H suppose the white Plane placed in order to receive

receive the Image of the Sun. Which that it may appear distinct and nicely terminated, 'tis necessary that the Rays which proceed from any one Point of the Sun, and which fall parallel upon the *Lens* A B, should again be collected in one Point upon the Plane. Wherefore the Distance between the *Lenses* A B and D ought to be something greater than in the common Disposition of the *Telescope*, or than when it is fitted for a good Sight; and the Position of the *Lens* B ought to be such, that the Rays which would otherwise tend to the *Focus* E of the *Lens* A B, may be diverted and brought to H; which may be done by *Prop.* XVI. or by taking E K, E D, E H in a continual Proportion. But in the common Disposition of the *Telescope*, the *Focus* K is required to coincide with the *Focus* E, as has been shewn above. So that here the Distance of the *Lenses* is increased by the Space E K, which will always be so much less, as the Distance E H is increased. For the Distance D K, which is the given *focal* Distance of the *Lens* D, is divided in such manner in E, that H E is to E D as the same E D to E K.

[*Fig.* 24, 26.] How great the Diameter of the Image of the Sun will appear upon the Plane H may thus be determined. Draw from the Center of the *Lens* A B to the *Lens* D the right Lines B P, B Q, comprehending

hending an Angle equal to that, under which the Sun's Diameter appears without a *Telescope*; and make BC a third Proportional to BK , BD , and join CP , CQ ; which will consequently give G the *Focus* of Rays diverging from B after Refraction at the *Lens* D ; which produce till they meet the Plane placed at H in the Points L , M . I say, LM will be the Diameter of the Sun's Image represented upon the Plane LHM . For produce PB , QB towards O and N . Therefore since from the extreme Point of the real Diameter of the Sun on the right Hand Side, Rays are sent upon the whole *Lens* AB , which are all to be looked upon as parallel among themselves, and to the right Line OB , one of them will proceed along the right Line OB , and penetrating the *Lens*, gon on in the right Line BP ; because B is the Center of the *Lens*, whose Thickness is here neglected. For the same Reason one of the parallel Rays from the left Extreme of the Sun's Diameter will proceed along the right Line NBQ . But moreover, both will be refracted in such manner by the *Lens* D , that diverging from their *Focus* C , they will proceed along the right Lines PL , QM , which are the right Lines CP , CQ produced. Therefore 'tis plain the Point in the right Hand Extreme of the Sun will be represented at L , and the opposite Point in the left Extreme at M .

For

For since the Image of the Sun is required distinct, it is necessary that where one Ray proceeding from any Point of it falls upon the Plane, all the rest which proceed from the same Point should be collected there also. Therefore the Diameter of the Image is LM : And by a *Telescope* made of a Convex and Concave, the Image is inverted; and by one made of two Convexes it is represented erect. *Q. E. D.*

But it must be observed, that the greater LM the Image of the Sun is, the *Lens's* AB and D remaining the same, the less clear and distinct will it be: For if all the Rays descending from the Sun upon the *Lens* AB , should possess a Space in the Place LHM equal to the Breadth of the *Lens* AB ; that is; if they were to form the Image of the Sun equal to the Aperture of the *Lens* AB , this Image would be as clear as if the Plane were enlightened by the Sun without the Interposition of *Lens's*. No respect being had to those Rays which the *Lens's* reflect, or by reason of their imperfect Transparency do not transmit, which perhaps occasions a Loss of above half the whole Number of Rays. But if the Sun's Image be made larger, which is necessary to be done in Observations of this nature, it will then be so much the more obscure. But Experience is the best Judge to determine

in what Magnitude it will be most convenient to represent the Sun's Image in these Observations; by trying first one, and then another Distance of the *Plane* from the *Telescope*. Where it is to be observed, that as we encrease this Distance, the Distance between the *Lens's* A B and D ought to be a little diminished, in order to preserve the Distinctness of the Image; the Reason of which has been already given.

W. B R O W N E.



A P P E N -



A P P E N D I X

TO this SECOND EDITION,

By J. T. DESAGULIERS, LL.D. and F.R.S.



THE first Thought we find in Print concerning a *Reflecting Telescope* is that of Dr. JAMES GREGORY, our Author's Uncle; who, in his *Optica promota* (Pag. 93 and 94) proposes a Catadioptrical Telescope (as being far preferable to Two other Sorts of Telescopes which he has been describing *) with no other View, but to make Telescopes shorter and more handy: For he and the rest of the World were at that time wholly unacquainted with the different Refrangibility of the Rays of Light; and consequently ignorant of the Errors occasioned in Refracting Telescopes by that Property of the Rays. Sir ISAAC NEWTON, the Discoverer of that wonderful *Phænomenon* in Light, being then also unacquainted with it; not having made any of his Experiments with the *Prism* till the beginning of the Year 1666.

* See his Words at the End of this *Appendix*, No. I.

Dr. J. GREGORY never brought this Telescope to any Degree of Perfection; but, owning his Want of Skill in Mechanics, only proposes it for others to execute. He had indeed an Object-*Speculum* of Metal ground to a Segment of a Sphere, and a little concave, as well as a little convex *Speculum*, ground by *Rives* and *Cox* (Optic-Glass-Grinders, famous in those Days) But because the Object Metal was not well polished, he only made some imperfect Trials, not so much as fitting the *Specula* and Eye-Glass into a Tube; and being discouraged, as much because he could not have the Object Metal ground into a parabolic Concave, as because that which he tried was not well polished, he gave over the Thoughts of bringing such Telescopes into use. (See his Letter to Mr. *Collins*, written from Saint *Andrezs*, Sept. 23, 1672. N^o. 4.)

But Mr. J. HADLEY, a few Years ago, after having made the *Newtonian* Reflecting Telescopes with good Success (the Description of which may be seen in N^o. 376 of the *Philosophical Transactions*) improving all Dr. J. GREGORY's Hints, did also bring this Telescope to Perfection by the Work of his own Hands; and has since taught his Majesty's Optician Mr. EDWARD SCARLET, and his Son, to make both the Sorts; which they do so well, that I have not yet known them exceeded in these Instruments.

struments by the Performance of any other Optic-Glass-Grinder.

To return to the History of the Reflecting Telescopes.

SIR ISAAC NEWTON, in the Year 1666, applied himself to grind Optic Glasses of other Figures than spherical (supposing, with all other Persons who had hitherto applied themselves to the Study of Optics, that Telescopes might be very much improved, by making use of Glasses ground to the Figure of some other conic Section; such as the parabolical, elliptical, or hyperbolical) but having made some Experiments with Prisms, whereby he discovered the different Refrangibility of the Rays of Light, he found that the Errors in Telescopes arising from that Cause was some hundreds of times greater than those which were occasioned by the spherical Figure of the Glasses, which did not collect the Rays into one Point, where Glasses of the Figure of a conic Section would do it if Light was uniform. This made him take Reflexions into Consideration; and finding them regular, so that the Angle of Reflexion of all sorts of Rays was equal to their Angle of Incidence, he understood that by their Mediation Optic Instruments might be brought to any Degree of Perfection imaginable, provided a *reflecting* Substance could be found, which would polish as finely as Glass, and *reflect*
as

as much Light as Glass *transmits*; and the Art of communicating to it a *parabolic* Figure be also attained. But he thought the Difficulties very great, and almost insuperable, when he further considered, that every Irregularity in a reflecting Superficies makes the Rays stray five or six times more out of their due Course, than the like Irregularities in a refracting one: So that a much greater Curiosity would be here requisite, than in figuring Glasses for Refraction.

Being then forced from *Cambridge* by the intervening Plague, he did not proceed any farther till two Years after.* Then, having considered what Dr. J. GREGORY proposed in his *Optica promota* concerning a *cata-dioptric* Telescope, with an Hole in the midst of the Object Metal to transmit the Light to an Eye-Glass placed behind it, he thought the Disadvantages would be so great, that he resolved, before he attempted any thing in the Practice, to alter Dr. GREGORY'S Design, and place the Eye-Glass at the Side of the Tube, rather than the Middle.† He then made two little Reflecting Telescopes with an Object Metal spherically concave; one of which he describes in the *Philosophical Transactions* (N^o. 80) and the other, which was better than the first, he sent to the Royal Society.

* *Phil. Transf.* No. 80. † *ib.* No. 83.

This last is described in a Letter to a Friend (see the *Originals* hereto subjoined, N^o. 2) but more fully in the said *Transactions*, N^o. 81; from which we here transcribe the Description, with Mr. HUYGENS's Remarks, and Sir ISAAC's Reply to them; as also a farther Account of the same Telescope, with a Table of *Apertures* and *Charges*, from the 82d *Transaction*.

An Account of a new Catadioptrical Telescope invented by Mr. NEWTON, Fellow of the Royal Society, and Professor of the Mathematics in the University of Cambridge.

“ This excellent Mathematician having
 “ given us, in the *Transactions* of *Febru-*
 “ *ary* last, an Account of the Cause which
 “ induced him to think of Reflecting Tele-
 “ scopes, instead of refracting ones, hath
 “ thereupon presented the curious World
 “ with an Essay of what may be performed
 “ by such Telescopes; by which it is found
 “ that Telescopical Tubes may be conside-
 “ rably shortened, without Prejudice to
 “ their magnifying Effect.

“ This new Instrument is composed of
 “ two metalline Speculums; the one con-
 “ cave, instead of an Object Glass, the other
 “ plane; and also of a small plano-convex
 “ Eye-Glass.

“ By

“ By *Fig. 2.* of *Tab. IV.* the Structure
 “ of it may be easily imagined; *viz.* That
 “ the Tube of this Telescope is open at the
 “ End which respects the Object; that the
 “ other End is close, where the said Con-
 “ cave is laid; and that near the open End
 “ there is a flat oval *Speculum*, made as
 “ small as may be, the less to obstruct the
 “ Entrance of the Rays of Light, and in-
 “ clined towards the upper Part of the
 “ Tube, where is a little Hole furnished
 “ with the said Eye-Glass: So that the
 “ Rays coming from the Object, do first
 “ fall on the Concave placed at the bottom
 “ of the Tube; and are thence reflected
 “ toward the other End of it, where they
 “ meet with the flat *Speculum*, obliquely
 “ posited; by the Reflexion of which they
 “ are directed to the little plano-convex
 “ Glass, and so to the Spectator's Eye,
 “ who looking downwards, sees the Object
 “ which the Telescope is turned to.

“ To understand this more distinctly and
 “ fully, the Reader may please to look
 “ upon the said Figure; in which

“ A B is the concave *Speculum*, of which
 “ the *Radius* or Semidiameter is $12\frac{2}{3}$ or
 “ 13 Inches.

“ C D, another metalline *Speculum*;
 “ whose Surface is flat, and the Circumfe-
 “ rence oval.

“ G D,

“ G D, an Iron Wire, holding a Ring
“ of Brass, in which the Speculum C D is
“ fixed.

“ F, a small Eye-Glass, flat above, and
“ convex below, of the twelfth part of an
“ Inch Radius, if not less; forasmuch as
“ the Metal collects the Sun's Rays at $6\frac{1}{3}$
“ Inches Distance, and the Eye-Glass at
“ less than $\frac{1}{6}$ of an Inch Distance from its
“ Vertex: Besides that the Author (as he
“ informs us) knew their Dimensions by the
“ Tools to (or in) which they were ground,
“ and particularly measuring the Diameter
“ of the hemi-spherical Concave, in which
“ the Eye-Glass was wrought, found it the
“ sixth part of an Inch.

“ G G G, the fore Part of the Tube fast-
“ ened to a Brass Ring H I, to keep it im-
“ moveable.

“ P Q K L, the hind Part of the Tube,
“ fastened to another Brass Ring P Q.

“ O, an Iron Hook fastened to the Ring
“ P Q, and furnished with a Skrew N,
“ thereby to advance or draw back the
“ hind Part of the Tube, and so by that
“ means to put the Specula in their due
“ Distance.

“ M Q G I, a crooked Iron sustaining the
“ Tube, and fastened by the Nail R to the
“ Ball and Socket S, whereby the Tube
“ may be turned every way.

E e

“ The

“ The Center of the flat Speculum, C D,
 “ must be placed in the same Point of the
 “ Tube’s Axe, where falls the Perpendicu-
 “ lar to this Axe, drawn to the same from
 “ the Center of the Eye-Glass; which
 “ Point is here marked at T.

And to give the Reader some Satisfaction to understand in what Degree it represents things distinct, and free from Colours, and to know the Aperture by which it admits Light; he may compare the Distances of the Focus E from the Vertex’s of the little Eye-Glass of the concave Speculum; that is, $EF \frac{1}{6}$ of an Inch, and $ETU 6\frac{1}{3}$ Inches; and the *Ratio* will be found as 1 to 38: Whereby it appears, that the Objects will be magnified about 38 times. To which Proportion is very consentaneous an Observation of the Crown on the Weather-cock about 200 Feet distant: For the Scheme X, *Fig. 3*, represents it bigger by $2\frac{1}{2}$ times in Diameter, when seen through this, than through an ordinary Telescope of about two Foot long.* And so supposing this ordinary one to magnify 13 or 14 times, as by the Description it should, this new one by the Experiment must magnify near as much as hath been assigned.

Thus far as to the Structure of this Telescope. Concerning the metalline Matter,

* *Fig. 4.*

fit for these reflecting Speculums, the Inventor has also considered the same; as may be seen by two of his Letters, written to the Publisher from *Cambridge, Jan. 18,* and 29, 167 $\frac{1}{2}$, to this Effect; *viz.*

1. That for a fit metalline Substance he would give this Caution; That whilst Men seek for a white, hard, and durable metalline Composition, they resolve not upon such an one as is full of small Pores, only discoverable by a Microscope: For though such an one may to appearance take a good Polish, yet the Edges of those small Pores will wear away faster in the polishing than the other Parts of the Metal; and so, however the Metal seem polite, yet it shall not reflect such with an accurate Regularity as it ought to do. Thus Tin-Glass mixed with ordinary Bell-Metal makes it more white, and apt to reflect a greater Quantity of Light; but withal its Fumes raised in the Fusion, like so many aerial Bubbles, fill the Metal full of those microscopical Pores. But white Arsenic both blanches the Metal, and leaves it solid without any such Pores, especially if the Fusion hath not been too violent. What the stellate *Regulus* of *Mars* (which I have sometimes used) or other such like Substance will do, deserves particular Examination.

To this he adds this further Intimation ; That Putty, or other such like Powder, with which it is polished, by the sharp Angles of its Particles fretteth the Metal, if it be not very fine, and fills it full of such small Holes as he speaketh of. Wherefore Care must be taken of that before Judgment be given, whether the Metal be throughout the Body of it porous or not.

2. He not having tried, as he saith, many Proportions of the Arsenic and Metal, does not affirm which is absolutely best ; but thinks there may conveniently be used any Quantity of Arsenic equalling in Weight between the sixth and eighth Part of the Copper, a greater Proportion making the Metal brittle.

The way which he used was this: He first melted the Copper alone, then put in the Arsenic ; which being melted, he stirred them a little together, bewaring in the mean time not to draw in Breath near the pernicious Fumes. After this he put in Tin ; and again so soon as that was melted (which was very suddenly) he stirred them well together, and immediately poured them off.

He saith, He knows not, whether by letting them stand longer on the Fire after the Tin was melted, a higher Degree of Fusion would have made the Metal porous ;
but

but he thought that way he proceeded to be safest.

He adds, That in that Metal which he sent to *London* there was no Arsenic, but a small Proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much Harm in making the Metal soft, as so less fit to be polished, as Good in rendering it white and luminous.

At another time he mixed Arsenic one Ounce, Copper six Ounces, and Tin two Ounces. And this an Acquaintance of his hath, as he intimates, polished better than he did the other.

As to the Objection, That with this kind of Prospectives Objects are difficultly found, he answers, in another Letter of his to the Publisher of *Jan. 6, 167 $\frac{1}{2}$* , That that is the Inconvenience of all Tubes that magnify much; and that after a little Use the Inconvenience will grow less, seeing that himself could readily enough find any Day-Objects, by knowing which way they were posited from other Objects that he accidentally saw in it; but in the Night to find Stars, he acknowledges it to be more troublesome; which yet may, in his Opinion, be easily remedied by two Sights affixed to the Iron Rod, by which the Tube is sustained; or by an ordinary Prospective-Glass, fastened

fastened to the same Frame with the Tube, and directed towards the same Object; as DES-CARTES in his *Dioptrics* hath described, for remedying the same Inconvenience of his best Telescopes.

So far the Inventor's Letters touching this Instrument: Of which having communicated the Description to Monf. CHRISTIAN HUYGENS *de ZULICHEM*, we received from him an Answer to this Effect, in his Letter of *Feb. 13, 1672, St. n.*

“ I see, by the Description you have sent
 “ me of Mr. NEWTON's admirable Tele-
 “ scope, that he hath well considered the
 “ Advantage which a concave Speculum
 “ hath above convex Glasses in collecting
 “ the parallel Rays; which certainly, ac-
 “ cording to the Calculation I have made
 “ thereof, is very great. Hence it is, that
 “ he can give a far greater Aperture to
 “ that Speculum, than to an Object-Glass
 “ of the same Distance of the *Focus*; and
 “ consequently, that he can much more
 “ magnify Objects that way, than by an
 “ ordinary Telescope. Besides, by it he
 “ avoids an Inconvenience, which is infe-
 “ parable from convex Object-Glasses, which
 “ is the Obliquity of both their Surfaces;
 “ which vitiateth the Refraction of the Rays
 “ that pass towards the Sides of the Glass,
 “ and does more Hurt than Men are aware
 “ of

“ of. Again; By the mere Reflexion of
 “ the metalline Speculum, there are not so
 “ many Rays lost as in Glasses; which re-
 “ flect a considerable Quantity by each of
 “ their Surfaces, and besides intercept many
 “ of them by the Obscurity of their Matter.
 “ Mean time, the main Business will be,
 “ to find a Matter for this Speculum that
 “ will bear so good and even a Polish as
 “ Glasses, and a Way of giving this Polish
 “ without vitiating the spherical Figure.
 “ Hitherto I have found no Specula that
 “ had near so good a Polish as Glass; and
 “ if M. NEWTON hath not already found
 “ a Way to make it better than ordinarily,
 “ I apprehend, his Telescopes will not so
 “ well distinguish Objects as those with
 “ Glasses. But 'tis worth while to search
 “ for a Remedy to this Inconvenience, and
 “ I despair not of finding one. I believe
 “ that M. NEWTON hath not been without
 “ considering the Advantage which a para-
 “ bolical Speculum would have above a
 “ spherical one in this Construction; but
 “ that he despairs, as well as I do, of work-
 “ ing other Surfaces than spherical ones
 “ with due Exactness; though else it be
 “ more easy to make a parabolical than el-
 “ liptical or hyperbolical one, by reason
 “ of a certain Property of the parabolic
 “ Conoid; which is, that all the Sections
 “ parallel

“ parallel to the Axis make the same Para-
 “ bola.

Thus far M. HUYGENIUS his judicious Letter ; to the latter Part of which, concerning the grinding parabolical Conoids, Mr. NEWTON saith, in his Letter to the Publisher of *Feb.* 20, 1671. That though he with him despairs of performing that Work by geometrical Rules, yet he doubts not but that the thing may in some measure be accomplished by mechanical Devices.

To all which I cannot but subjoin an Extract of a Letter received very lately (*March* 19) from the Inventor of this new Telescope, from *Cambridge* ; viz.

“ In my last Letter I gave you Occasion
 “ to suspect, that the Instrument which I
 “ sent you is, in some respect or other, in-
 “ disposed, or that the Metals are tarnish-
 “ ed ; and by your Letter of *March* 16,
 “ I am fully confirmed in that Opinion : For,
 “ whilst I had it, it represented the Moon
 “ in some Parts of it as distinctly as other
 “ Telescopes usually do which magnify as
 “ much as that. Yet I very well know,
 “ that that Instrument hath its Imperfec-
 “ tions, both in the Composition of the
 “ Metal, and in its being badly cast ; as
 “ you may perceive by a scabrous Place
 “ near the middle of the Metal of it on the
 “ polished Side, and also in the Figure of
 “ that

“ that Metal near that scabrous Place. And
 “ in all those Respects that Instrument is
 “ capable of further Improvement.

“ You seem to insinuate, that the Pro-
 “ portion of 38 to 1 holds only for its
 “ magnifying Objects at small Distances.
 “ But if for such Distances, suppose 500
 “ Feet, it magnifies at that rate, by the
 “ Rules of Optics it must for the greatest
 “ Distance imaginable magnify more, than
 “ $37\frac{3}{4}$ to 1; which is so considerable a di-
 “ minishing, that it may be even then as
 “ 38 to 1.

“ Here is made another Instrument like
 “ the former, which does very well. Ye-
 “ sterday I compared it with a six Foot
 “ Telescope, and found it not only to mag-
 “ nify more, but also more distinctly. And
 “ to day I found that I could read in one of
 “ the *Philosophical Transactions*, placed
 “ in the Sun's Light, at an hundred Feet
 “ Distance; and that at an hundred and
 “ twenty Feet Distance I could discern some
 “ of the Words. When I made this Trial,
 “ its Aperture (defined next the Eye) was
 “ equivalent to more than an Inch and a
 “ third part of the Object-Metal. This
 “ may be of some Use to those that shall
 “ endeavour any thing in Reflexions; for
 “ hereby they will in some measure be en-
 “ abled to judge of the Goodness of their
 “ Instruments, &c.

*Mr. NEWTON's Letter to the Publisher, of
March 26, 1672, containing some more
Suggestions about his new Telescope;
and a Table of Apertures and Charges
for the several Lengths of that Instru-
ments.*

“ S I R,

“ Since my last Letter I have further
“ compared the two Telescopes, and find
“ that of Metal to represent as well the
“ Moon as nearer Objects, something dis-
“ tincter than the other. But I must tell
“ you also, that I am not very well assured
“ of the Goodness of that other, which I
“ borrowed to make the Comparison; and
“ therefore desire that the other Experiment
“ should be rather confided in, of reading
“ at the Distance of between 100 and 120
“ Feet, at which I and others could read
“ with it in the *Transactions*, as I found
“ by Measure: At which time the Aperture
“ was $1\frac{1}{3}$ of an Inch; which I knew by
“ trying, that an Obstacle of that Breadth
“ was requisite to intercept all the Light
“ which came from one Point of the Ob-
“ ject.

“ I should tell you also, that the little
“ plane Piece of Metal near the Eye-Glass
“ is not truly figured; whereby it happens,
“ that Objects are not so distinct at the
“ Middle

“ Middle as at the Edges. And I hope,
 “ that by correcting its Figure (in which
 “ I find more Difficulty than one would ex-
 “ pect) they will appear all over distinct,
 “ and distincter in the Middle than at the
 “ Edges. And I doubt not but that the
 “ Performances will then be greater.

“ But yet I find, that there is more Light
 “ lost by Reflexion of the Metal, which I
 “ have hitherto used, than by Transmission
 “ through Glasses: For which Reason a
 “ shallower Charge would probably do
 “ better for obscure Objects; suppose such
 “ an one as would make it magnify 34 or
 “ 32 times. But for bright Objects at any
 “ Distance, it seems capable of magnifying
 “ 38 or 40 times with sufficient Distinct-
 “ ness. And for all Objects the same
 “ Charge, I believe, may, with Advantage,
 “ be allowed, if the steely Matter, imploy-
 “ ed at *London*, be more strongly reflec-
 “ tive than this which I have used.

“ The Performances of one of these In-
 “ struments of any Length being known,
 “ it will appear by this following Table
 “ what may be expected from those of other
 “ Lengths by this Way, if Art can accom-
 “ plish what is promised by the Theory.
 “ In the first Column is expressed the Length
 “ of the Telescope in Feet; which doubled

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“ gives the Semidiameter of the Sphere,
 “ on which the concave Metal is to be
 “ ground.

Lengths.	Apertures.	Charges.
$\frac{1}{2}$	100	100
1	168	119
2	283	141
3	383	157
4	476	168
5	562	178
6	645	186
8	800	200
10	946	211
12	1084	221
16	1345	238
20	1591	254
24	1824	263

“ The Use of this Table will best ap-
 “ pear by Example: Suppose therefore a
 “ half Foot Telescope may distinctly mag-
 “ nify 30 times with an Inch Aperture,
 “ and it being required to know what
 “ ought to be the analogous Constitution
 “ and Performance of a four Foot Tele-
 “ scope; by the second Column, as 100 to
 “ 476, so are the Apertures, as also the
 “ Number of times which they magnify.
 “ And consequently, since the half Foot
 “ Tube hath an Inch Aperture, and magni-
 “ fieth

“ fieth 30 times; a four Foot Tube pro-
 “ portionally should have $4\frac{7}{10}$ Inches A-
 “ perture, and magnify 143 times. And
 “ by the third Column, as 100 to 168, so
 “ are their Charges. And therefore if the
 “ Diameter of the Convexity of the Eye-
 “ Glass for a half Foot Telescope be $\frac{1}{3}$ of
 “ an Inch, that for a four Foot should be
 “ $\frac{1}{3}\frac{6}{10}\frac{8}{10}$; that is, about $\frac{1}{3}$ of an Inch.

“ In like manner, if a half Foot Tele-
 “ scope may distinctly magnify 36 times
 “ with $1\frac{1}{4}$ of an Inch Aperture, a four
 “ Foot Telescope should with equal Dis-
 “ tinctness magnify 171 times with 6 Inches
 “ Aperture; and one of six Foot should
 “ magnify 232 times with $8\frac{2}{3}$ Inches Aper-
 “ ture; and so of other Lengths. But
 “ what the Event will really be, we must
 “ wait to see determined by Experience:
 “ Only this I thought fit to insinuate, that
 “ they which intend to make Trials in
 “ other Lengths, may more readily know
 “ how to design their Instruments. Thus
 “ for a four Foot Tube, since the Aperture
 “ should be five or 6 Inches, there will be
 “ required a Piece of Metal 7 or 8 Inches
 “ broad at least; because the Figure will
 “ scarcely be true to the Edges: And the
 “ Thickness of the Metal must be propor-
 “ tional to the Breadth, lest it bend in the
 “ grinding. The Metals being polished,
 “ there

“ there may be Trials made with several
 “ Eye-Glasses, to find what Charge may
 “ with best Advantage be made use of.

*An Extract of another Letter of the same
 to the Publisher, dated March 30, 1672.
 by way of Answer to some Objections
 made by an ingenious French Philosopher
 to the new Reflecting Telescope.*

“ S I R,

“ I doubt not but M. A. will allow the
 “ Advantage of Reflexion in the Theory
 “ to be very great, when he shall have in-
 “ formed himself of the different Refrangi-
 “ bility of the several Rays of Light. And
 “ for the practic Part, it is in some mea-
 “ sure manifest by the Instruments already
 “ made, to what Degree of Vivacity and
 “ Brightness a metalline Substance may be
 “ polished. Nor is it improbable but that
 “ there may be new Ways of Polishing
 “ found out for Metal, which will far ex-
 “ cel those that are yet in use. And when
 “ a Metal is once well polished, it will be
 “ a long time preserved from tarnishing,
 “ if Diligence be used to keep it dry, and
 “ close shut up from Air : For the principal
 “ Cause of tarnishing seems to be, the con-
 “ densing of Moisture on its polished Sur-
 “ face ; which by an acid Spirit, where-
 “ with

“ with the Atmosphere is impregnated,
 “ corrodes and rusts it ; or at least, at its
 “ exhaling, leaves it covered over with a
 “ thin Skin, consisting partly of an earthly
 “ Sediment of that Moisture, and partly of
 “ the Dust, which flying to and fro in the
 “ Air, had settled and adhered on it.

“ When there is not Occasion to make
 “ frequent Use of the Instrument, there
 “ may be other Ways to preserve the Metal
 “ for a long time ; as perhaps by immer-
 “ ging it in Spirit of Wine, or some other
 “ convenient Liquor. And if they chance
 “ to tarnish, yet their Polish may be reco-
 “ vered by rubbing them with a soft Piece
 “ of Leather, or other tender Substance,
 “ without the Assistance of any fretting
 “ Powders, unless they happen to be rusty ;
 “ for then they must be new polished.

“ I am very sensible, that Metal reflects
 “ less Light than Glass transmits ; and for
 “ that Inconvenience I gave you a Remedy
 “ in my last Letter, by assigning a shallow-
 “ er Charge, in proportion to the Aperture,
 “ than is used in other Telescopes. But, as
 “ I have found some metalline Substances to
 “ be more strongly reflective, and to polish
 “ better, and be freer from tarnishing,
 “ than others ; so I hope there may in time
 “ be found out some Substance much freer
 “ from these Inconveniences than any yet
 “ known,

After

After Sir ISAAC NEWTON'S Telescope had been sent up to the Royal Society, they gave Mr. COX (the Optic-Glass-Grinder) Orders to make one after the same manner of Contrivance four Foot long; which was done, one End of the Tube being open, and at the other End was placed a concave metalline Mirrour; the Diameter of which was betwixt 4 and 5 Inches; and it was ground on a Sphere of 14 Foot Diameter; and about its Focus, which was about 4 Foot off, was placed a reflecting Plate as big as a Two-pence, inclined at an Angle of 45 Degrees to the Axis: So that the reflected Rays falling thereon were again reflected upright to the Side of the Telescope; where the Eye, through a small Hole wherein is placed a small plano-convex Glass, beholds the Object on the reflecting Plate as much magnified as it could have been done by an ordinary Telescope of 40 Foot long or more, and void of Colours. The Mirrour and reflecting Plate were made to be taken out and wiped at pleasure. But the Society and COX himself were not pleased with the Metal or Polish of the reflecting Plate; and therefore a Trial was to be made with the *Lapis Osmandinus*, a black Stone that comes from Mount *Hecla* in *Island*, and other Materials, &c. (*Whether or no these Trials succeeded, I have not been able*

to

to learn; but I believe they did not, nor any other, till Mr. HADLEY made his Newtonian Telescope in 1723.) This Account I copied from a Paper of Mr. JOHN COLLINS; in which was the Copy of Sir ISAAC NEWTON's Letter to a Friend about his Telescope (*Orig. Pap. N^o. 2.*)

A little after Sir ISAAC NEWTON had sent his new Telescope to the Royal Society, Mr. OLDENBURGH, the Secretary, wrote him a Letter of Thanks; to which Sir ISAAC made Answer in 167 $\frac{1}{2}$, giving a farther Account of the Instrument. (*Orig. Pap. N^o. 5.*)

About this time Dr. J. GREGORY having an Account of Sir ISAAC NEWTON's Telescope, wrote his Thoughts about it to Mr. JOHN COLLINS, in a Letter from *Aberdeen*, dated *Aug. 6, 1672*; in which he gives the Preference to Sir ISAAC's Telescope above that which he described in his *Optica promota* in one respect, but thinks his own better in another. (*Orig. Pap. N^o. 3.*) At the same time one Monf. CASSEGRAIN published a Description of a Catadioptric Telescope, as his own Invention; which he pretended to have been prior to Sir ISAAC's Telescope, and which M. BERCE, Publisher of the *French Memoirs* for the Year 1672, seems to prefer to it: But S. SALVETTI, one of the Great Duke's Musicians at *Florence*,

G g

who

who made one after the manner of Sir ISAAC, thinks it much better contrived than M. CASSEGRAIN'S. (*Phil. Trans.* N^o. 87.) Now M. CASSEGRAIN'S is not pretended to have been contrived before the Beginning of the Year 1672, and Sir ISAAC'S was contrived in the Year 1666, and executed in the Year 1670, or at farthest finished in the Year 1671: Besides, M. CASSEGRAIN'S differs in nothing from Dr. GREGORY'S, but that he would have the small Metal to be convex, which Dr. GREGORY makes concave; and therefore the Instrument seems only to be Dr. GREGORY'S disguised. The whole Account of this, and Sir ISAAC NEWTON'S Thoughts upon it, I here subjoin, as copied from the *Philos. Transactions*, N^o. 83.

Mr. ISAAC NEWTON'S Considerations on Part of a Letter of Monsieur de BERCE, printed in the eighth French Memoire, concerning the Catadioptrical Telescope pretended to be improved and refined by M. CASSEGRAIN.

THAT the Reader may be enabled the better to judge of the whole, by comparing together the Contrivances both of Mr. NEWTON and M. CASSEGRAIN, it will be necessary to borrow from the said *French Memoire* what is there said concerning them; which is as follows.

“ I send

“ I send you (*saitb* M. de BERCE *to the*
Publisher of the Memoire) the Copy of the
 “ Letter which M. CASSEGRAIN hath
 “ written to me, concerning the Propor-
 “ tions of Sir SAMUEL MORELAND’s Trum-
 “ pet. And as for the Telescope of Mr.
 “ NEWTON, it hath as much surprized me
 “ as the same Person that hath found out
 “ the Proportions of the Trumpet: For it
 “ is now about three Months that that Per-
 “ son communicated to me the Figure of a
 “ Telescope, which was almost like it, and
 “ which he had invented; but which I
 “ look upon as *more witty*.* I shall here
 “ give you the Description of it in short.
 “ A B C D (*Plate IV. Fig. 5.*) is a strong
 “ Tube, in the Bottom of which there is a
 “ great concave *Speculum* C D, pierced in
 “ the Middle E.

“ F is a convex *Speculum*, so disposed,
 “ as to its Convexity, that it reflects the
 “ Species which it receives from the great
 “ *Speculum* towards the Hole E, where is
 “ an Eye-Glass which one looketh through.

“ The Advantages which I find in this
 “ Instrument above that of Mr. NEWTON,
 “ are, 1. That the Mouth or Aperture A B
 “ of the Tube may be of what Bigness you
 “ please; and consequently, you may have

* *More ingenious*, is the Sense of the *French*.

“ many more Rays upon the concave *Specu-*
 “ *lum*, than upon that of which you have
 “ given us the Description. 2. The Re-
 “ flexion of the Rays will be very natural;
 “ since it will be made upon the Axis it
 “ self, and therefore more vivid. 3. The
 “ Vision of it will be so much the more
 “ pleasing; in that you shall not be incom-
 “ moded by the great Light, by reason of
 “ the Bottom CD, which hideth the whole
 “ Face. Besides that you’ll have less Dif-
 “ ficulty in discovering the Objects, than
 “ in that of Mr. NEWTON.

So far this *French* Author. To which
 we shall now subjoin the Considerations of
 Mr. NEWTON, as we received them from
 him in a Letter written from *Cambridge*;
May 4, 1672. as follows.

S I R,

“ I should be very glad to meet with any
 “ Improvement of the Catadioptrical Tele-
 “ scope; but that Design of it, which (as
 “ you inform me) Mr. CASSEGRAIN hath
 “ communicated three Months since, and
 “ is now printed in one of the *French Me-*
 “ *moires*, I fear will not answer Expecta-
 “ tion: For, when I first applied myself
 “ to try the Effects of Reflexion, Mr.
 “ GREGORY’s *Optica promota* (printed
 “ in

“ in the Year 1663) being fallen in my
 “ Hands, where there is an Instrument
 “ (described Page 94) like that of Mons.
 “ CASSEGRAIN’S, with a Hole in the midst
 “ of the Object-Metal to transmit the Light
 “ to an Eye-Glass placed behind it ; I had
 “ thence an Occasion of considering that
 “ Sort of Constructions, and found their
 “ Disadvantages so great, that I saw it ne-
 “ cessary, before I attempted any thing in
 “ the Practice, to alter the Design of them,
 “ and place the Eye-Glass at the Side of
 “ the Tube, rather than at the Middle.

“ The Disadvantages of it you will un-
 “ derstand by these Particulars: 1. There
 “ will be more Light lost in the Metal by
 “ Reflexion from the little convex *Specu-*
 “ *lum*, than from the oval Plane: For it
 “ is an obvious Observation, that Light is
 “ most copiously reflected from any Sub-
 “ stance when incident most obliquely.
 “ 2. The convex *Speculum* will not reflect
 “ the Rays so truly as the oval Plane, un-
 “ less it be of an hyperbolic Figure ; which
 “ is incomparably more difficult to form
 “ than a plane ; and if truly formed, yet
 “ would only reflect those Rays truly which
 “ respect the *Axis*. 3. The Errors of the
 “ said Convex will be much augmented by
 “ the too great Distance through which the
 “ Rays reflected from it must pass, before
 “ their

“ their Arrival at the Eye-Glaſs. For
 “ which Reaſon I find it convenient to
 “ make the Tube no wider than is neceſſa-
 “ ry, that the Eye-Glaſs be placed as near
 “ to the oval Plane as is poſſible, without
 “ obſtructing any uſeful Light in its Paſſage
 “ to the Object-Metal. 4. The Errors of
 “ the Object-Metal will be more augment-
 “ ed by Reflexion from the Convex than
 “ from the Plane, becauſe of the Inclination
 “ or Deflexion of the Convex on all Sides
 “ from the Points on which every Ray
 “ ought to be incident. 5. For theſe Rea-
 “ ſons there is requiſite an extraordinary
 “ Exactneſs in the Figure of the little Con-
 “ vex; whereeas I find by Experience, that
 “ it is much more difficult to communicate
 “ an exact Figure to ſuch ſmall Pieces of
 “ Metal, than to thoſe that are greater.
 “ 6. Becauſe the Errors at the Perimeter
 “ of the concave Object-Metal, cauſed by
 “ the Sphericalneſs of its Figure, are much
 “ augmented by the Convex, it will not
 “ with Diſtinctneſs bear ſo large an Aper-
 “ ture as in the other Conſtructions. 7. By
 “ reaſon that the little Convex conduces
 “ very much to the magnifying Virtue of
 “ the Inſtrument, which the oval Plane
 “ doth not, it will magnify much more in
 “ proportion to the Sphere on which the
 “ great Concave is ground, than in the
 “ other

“ other Design ; and so magnifying Objects
 “ much more than it ought to do in pro-
 “ portion to its Aperture, it must represent
 “ them very obscure and dark ; and not on-
 “ ly so, but also confused, by reason of its
 “ being overcharged. Nor is there any
 “ convenient Remedy for this : For if the
 “ little Convex be made of a larger Sphere,
 “ that will cause a greater Inconvenience,
 “ by intercepting too many of the best
 “ Rays ; or, if the Charge of the Eye-
 “ Glass be made so much shallower, as is
 “ necessary, the Angle of Vision will there-
 “ by become so little, that it will be very
 “ difficult and troublesome to find an Ob-
 “ ject ; and of that Object, when found,
 “ there will be but a very small Part seen
 “ at once.

“ By this you may perceive, that the
 “ three Advantages which Monf. C A S S E-
 “ G R A I N propounds to himself, are rather
 “ Disadvantages : For, according to his
 “ Design, the Aperture of the Instrument
 “ will be but small, the Object dark and
 “ confused, and also difficult to be found.
 “ Nor do I see why the Reflexion is more
 “ upon the same *Axis*, and so more natural
 “ in the one Case than in the other ; since
 “ the *Axis* itself is reflected towards the
 “ Eye by the oval Plane ; and the Eye
 “ may be defended from external Light as
 “ well

“ well at the Side as at the Bottom of the
 “ Tube.

“ You see therefore that the Advantages
 “ of this Design are none, but the Disad-
 “ vantages so great and unavoidable, that
 “ I fear it will never be put in Practice with
 “ good Effect. And when I consider, that
 “ by reason of its Resemblance with other
 “ Telescopes, it is something more obvious
 “ than the other Construction, I am apt
 “ to believe that those who have attempted
 “ any thing in Catoptrics, have ever tried
 “ it in the first place; and that their bad
 “ Success in that Attempt hath been the
 “ Cause why nothing hath been done in
 “ Reflexions: For Mr. GREGORY speaking
 “ of these Instruments in the aforesaid Book,
 “ Page 95. saith, *De Mechanica horum*
 “ *Speculorum & Lentium ab aliis frustra*
 “ *tentatâ, ego in Mechanicis minus versa-*
 “ *tus nihil dico.* So that there have been
 “ Trials made of these Telescopes, but yet
 “ in vain. And I am informed that about
 “ seven or eight Years since, Mr. GREGORY
 “ himself, at *London*, caused one of six
 “ Foot to be made by Mr. *Reive*; which I
 “ take to have been according to the afore-
 “ said Design described in his Book, be-
 “ cause though made by a skilful Artist, yet
 “ it was without Success.

“ I could

“ I could wish therefore Mr. CASSEGRAIN
 “ had tried his Design before he divulged
 “ it: But if, for further Satisfaction, he
 “ please hereafter to try it, I believe the
 “ Success will inform him, that such Pro-
 “ jects are of little Moment till they be put
 “ in Practice.

After this Dr. GREGORY, in a Letter to
 Mr. COLLINS from St. *Andrews*, Sept. 23.
 1672 (N^o. 4.) replies to Sir ISAAC'S Ani-
 madversions upon M. CASSEGRAIN'S Tele-
 scope, looking upon it as his own disguised;
 and proposes to use a plane *Speculum* instead
 of his concave or CASSEGRAIN'S convex, to
 remedy some Inconveniencies found by Sir
 ISAAC, and make his Telescope still shorter
 than Sir ISAAC'S. He thinks the oblique
 Reflexion from the little Plate in Sir ISAAC'S
 much worse than his own direct Reflexion
 from his small *Speculum*; but thinks a little
 Concave or Convex worth trying, because
 different Charges may be then given to the
 Telescope with the same Eye-Glass; which
 he thinks impracticable in Sir ISAAC'S. Sir
 ISAAC'S Answer to these Objections was sent
 to Mr. COLLINS, which he communicated
 to Mr. GREGORY (*but I could not meet with*
that Letter of Sir ISAAC'S) who seems there-
 by convinced, that an oblique Reflexion is
 preferable to a direct one; but does not con-
 H h ceive

ceive how Sir ISAAC can alter the Charge of his Telescope without changing the Eye-Glass.

(N. B. *Dr. GREGORY'S Letter which contains this Reply is printed at the End of the Appendix, No. 6. But I don't take notice here of the Arguments pro and con concerning the Tubercula, or little Eminences in the reflecting small Metal being struck directly or obliquely by the Rays of Light; because in fact they are not struck at all by the Rays of Light, which are reflected without Contact or Impulsion against the Tubercula, or any Part of the polished Metal; as Sir ISAAC found out afterwards, though he did not know it then.*)

Sir ISAAC in his next Letter (the last we publish here of his, N^o. 7.) gives an Account of his Manner of varying the Charge in his Telescope, by making use of a Glass Prism with two convex Surfaces and a flat one, instead of his small plane Mirrour. He also shews how the Aperture of the Eye-Glass ought to be limited in Dr. GREGORY'S Telescope; and for his own Telescope he lays all the Stress of Magnifying upon the Eye-Glass. The rest of the Letter is concerning a Reflecting Microscope mentioned by him in the *Transactions*, and a reflecting Glass Mirrour for burning, proposed

posed by Dr. GREGORY in one of his Letters (N^o. 6.) which Mirrours are now very commonly made.

Dr. GREGORY in his last Letter to Mr. COLLINS on this Subject, from St. *Andrews*, May 13, 1673. (N^o. 8.) commends Sir ISAAC NEWTON's Way of varying the Charge of his Telescope; but thinks it liable to some Errors, owing to the Refraction of the Rays at their Entrance into and Emerfion out of the convex Surfaces of the little Prism. He thinks it not worth while to look on terrestrial Bodies with excellent Telescopes, because they magnify the Particles of the Atmosphere as well as the Object. Then he proposes a Plate of Metal with a small Hole in it to be placed in the *Focus* of his Eye-Glass next to the Eye, to intercept all the spurious Rays (*without which; as it has appeared since in Practice, this Telescope would be of no Use*) and concludes with asserting, that if it appears by Trials that common Microscopes (that is Microscopes with two or three Glasses) can be brought to exceed all Improvements of a single *Lens*, then it will follow from thence that his Telescope may be brought to exceed Sir ISAAC NEWTON's.

There have been no Attempts made since that time (*viz.* since the Year 1673.) to

make either of these Sorts of Telescopes,* at least no successful ones that have been made publick, till in the Year 1723, that is at the Distance of fifty Years, Mr. HADLEY made the reflecting Telescope described in the *Philosophical Transactions*, N^o. 376. to which I refer the Reader: But I think it not improper to copy from another *Transaction* (N^o. 378.) the Observations made by the late Reverend Mr. POUND, and his Nephew the Reverend Mr. BRADLEY, at *Wanstead*, in the Year 1723. as also others made by Mr. HADLEY himself with this Telescope, before I give an Account also of the *Gregorian* Telescope, now likewise made and brought to Perfection by the said Mr. HADLEY, together with a Table of his Calculations concerning this Telescope, communicated to me lately.

Phil. Trans. N^o. 378. *Pag.* 382.

A Letter from the Rev. Mr. JAMES POUND, Rector of Wanstead, F. R. S. to Dr. JURIN, Secr. R. S. concerning Observations made with Mr. HADLEY's Reflecting Telescope.

* Three or four *French* Authors have indeed mentioned the Use of concave metalline Mirrours, instead of one of the Eye-Glasses of a Dioptrical Telescope; but the Thing was never put in Practice.

“ It

“ It were to be wished, that, with the
 “ particular Description given in a late
 “ *Transaction* (N^o. 376.) of the curious
 “ Mechanism of that Catadioptric Tele-
 “ scope which was made by Mr. HADLEY,
 “ and by him presented to the *Royal So-*
 “ *ciety*, that most ingenious Gentleman
 “ would have communicated also a full Ac-
 “ count of what Observations he had made
 “ with it; whereby the Publick might at
 “ length have been apprized of the Usefulness
 “ of an Invention (worthy of its great
 “ Author Sir ISAAC NEWTON) which,
 “ perhaps from the vain Attempts made by
 “ some of putting it in Practice, hath lain
 “ neglected these fifty Years: For it is so
 “ long since it was first published in the
 “ *Philosophical Transactions*, N^o. 81.

“ Mr. HADLEY hath sufficiently convin-
 “ ced us, that this noble Invention doth not
 “ consist in bare Theory; and it is to be
 “ hoped, that he, or some other such cu-
 “ rious and worthy Persons (who scruple
 “ not at a little Pains and Cost) will in a
 “ short time find out a Method, either of
 “ preserving the concave Metal from tar-
 “ nishing, or of clearing it easily when tar-
 “ nished, or else of making a good concave
 “ *Speculum* of Glass quicksilvered on the
 “ back part. When a Method for either
 “ of these shall be discovered, 'tis not to be
 “ doubted,

“ doubted, but that the old Dioptric Te-
 “ lescope will be for the most part laid by,
 “ and this Catoptric one will be chiefly in
 “ use among the practical Astronomers; in-
 “ asmuch as several Inconveniences and
 “ Difficulties, which are unavoidable in
 “ the Management of the former, especial-
 “ ly when long, are in this latter wholly
 “ avoided.

“ It is no small Convenience, that, by
 “ means of one of these Reflecting Tele-
 “ scopes, whose Length exceeds not five
 “ Feet (and which may be managed at a
 “ Window within the House) celestial Ob-
 “ jects appear as much magnified, and as
 “ distinct, as they do through the common
 “ Telescope of more than a hundred Feet
 “ in Length.

“ Mr. BRADLEY, the *Savilian* Professor
 “ of Astronomy, and myself, have compared
 “ Mr. HADLEY's Telescope (in which the
 “ focal Length of the Object-Metal is not
 “ quite 5 Feet and $\frac{1}{4}$) with the *Huygenian*
 “ Telescope, the focal Length of whose
 “ Object-Glass is 123 Feet: And we find,
 “ that the former will bear such a Charge,
 “ as to make it magnify the Object as many
 “ times as the latter with its due Charge;
 “ and that it represents Objects as distinct,
 “ tho' not altogether so clear and bright;
 “ which may be occasioned partly from the
 “ Diffe-

“ Difference of their Apertures (that of the
 “ *Huygenian* being somewhat the larger)
 “ and partly from several little Spots in the
 “ concave Surface of the Object-Metal,
 “ which did not admit of a good Polish.

“ Notwithstanding this Difference in the
 “ Brightness of the Objects, we were able;
 “ with this Reflecting Telescope, to see
 “ whatever we have hitherto discovered
 “ by the *Huygenian*; particularly the Tran-
 “ sits of *Jupiter's* Satellites, and their
 “ Shades, over the Disk of *Jupiter*; the
 “ black List in *Saturn's* Ring, and the
 “ Edge of the Shade of *Saturn* cast on his
 “ Ring, as represented by *Fig. 4. Plate*
 “ II. of the forementioned *Transaction*,
 “ N^o. 376.

“ We have also seen with it several
 “ times the five Satellites of *Saturn*; in
 “ viewing of which this Telescope had the
 “ Advantage of the *Huygenian*, at that
 “ time when we compared them: For it be-
 “ ing in Summer, and the *Huygenian* Te-
 “ lescope being managed without a Tube,
 “ the Twilight prevented us from seeing in
 “ this some of those small Objects, which
 “ at the same time we could discern with
 “ the reflecting Telescope.

“ *I am, &c.*

“ J A. P O U N D.

Observe-

Observations on the Satellites of Jupiter and Saturn, made with the same Telescope. By JOHN HADLEY, Esq; F. R. S. Extracted from the Minutes of the Royal Society, April 6. 1721.

“ Mr. HADLEY gave the *Society* a Relation of some of the most remarkable Observations, which he had made with his Reflecting Telescope, before he presented it to the *Society*.

“ In observing *Jupiter's* Satellites, he has seen distinctly the Shadows of the first and third Satellites cast upon the Body of the Planet: Mr. FOLKES and Dr. JURIN, being present, affirmed, that Mr. HADLEY had likewise shewn them the Shadow of the third Satellite through the same Telescope.

“ In observing *Saturn* the last Spring, at a Time when that Planet was about 15 Days past the Opposition, he saw the Shade of the Planet cast upon the Ring, and plainly discerned the Ring to be distinguished into two Parts, by a dark Line, concentric to the Circumference of the Ring: The outer or upper Part of the Ring seemed to be narrower than the lower or inner Part, next the Body; and the dark Line, which separated them,

“ was

“ was stronger next the Body, and fainter
 “ on the outer Part towards the upper
 “ Edge of the Ring. Within the Ring he
 “ discerned two Belts; one of which crossed
 “ *Saturn* close to its inner Edge, and seem-
 “ ed like the Shade of the Ring upon the
 “ Body of *Saturn*: But when he consider-
 “ ed the Situation of the Sun, in respect to
 “ the Ring and *Saturn*, he found that Belt
 “ could not arise from such a Cause.

“ He says, that at Times he has seen with
 “ this Telescope three different Satellites
 “ of *Saturn*; but could never have the
 “ Fortune to see all five.

“ *Aug.* 1723. Mr. HADLEY adds, that
 “ he has several times seen the Shadow of
 “ the first, second, and third Satellites of
 “ *Jupiter* pass over the Body of that Pla-
 “ net; and that he has seen the first and
 “ second appear, as a bright Spot upon the
 “ Body of *Jupiter*, and has been able to
 “ keep Sight of them there for about a
 “ Quarter of an Hour, from the Time of
 “ their entering on his Limb.

“ *Jupiter*’s Satellites have of late Years
 “ been so situated, with regard to the
 “ Earth and *Jupiter*, that he has not had
 “ sufficient Opportunity of observing the
 “ Transit of the fourth Satellite, or of its
 “ Shadow.

“ The dark Line on the Ring of *Saturn*,
 “ parallel to its Circumference, is chiefly
 “ visible on the *Anse*, or Extremities of the
 “ elliptic Figure, in which the Ring ap-
 “ pears ; but he has several times been able
 “ to trace it very near, if not quite round ;
 “ particularly in *May* 1722. he could dis-
 “ cern it without the Northern Limb of
 “ *Saturn*, in that Part of the Ring that
 “ appeared beyond the Globe of the Planet.
 “ The Globe of *Saturn* (at least towards
 “ its Limb) reflects less Light than the in-
 “ ner Part of the Ring ; and he has some-
 “ times distinguished it from the Ring by
 “ the Difference of Colour.
 “ The dusky Line, which in 1720 he
 “ observed to accompany the inner Edge of
 “ the Ring cross the Disk, continues close
 “ to the same, though the Breadth of the
 “ Ellipse is considerably increased since
 “ that time.

An Account of the *Gregorian Reflecting*
Telescope, as perfected, by JOHN HADLEY,
 Esq; *Vice-President* of the *Royal So-*
ciety, in the Year 1726.

At the Bottom of the large Tube (ex-
 pressed by a strong black Line) is fixed the
 concave metalline *Speculum* B B, perforated
 with the Hole C C ; opposite to which
 Hole

Hole is placed a small *Speculum* of the same Metal FF concave towards the great Metal, and so fixed to a crooked Arm, that it may be brought towards or carried from-wards the great *Speculum*, keeping its *Axis* still in the same Line (*viz.* in the common *Axis* of both *Specula*) and by that means parallel Rays, or Rays from the Points of a very distant Object, coming to the great *Speculum* in the Lines OO, PP, &c. and falling upon the great *Speculum* between B and C will be so united at its *Focus*, as to form there the Image GG of the Object OP supposed at a vast Distance. The Rays diverging again from their respective Points of the Image, go on diverging, and fall upon the little Concave FF, whose focal Length is DI, and from its Surface are reflected nearly parallel to their respective *Axes* (not wholly so, because DG is greater than DI) and with all the *Axes* or principal Rays move parallel to the common *Axis* through the Hole in the great *Speculum*, in the Direction DA, so entering into the small Tube N M M N, which is fixed to the great Tube behind the *Speculum*, fall upon the convex Side of the plano-convex ocular *Lens* NN, and passing through it, form a second Image at gg, whose Bigness is limited by the Hole of the perforated opake Circle or Diaphragm

placed at R R. That second or erect Image of the first inverted Image of an erect Object is seen large, clear, and distinct by the Eye at E, which sees it through the small Hole in the Plate M M, and the last Eye-Glass S S, which is a *Meniscus*: For the Eye will see it under the Angle S E S made by the *Axes* of those Pencils of Rays which came from the Extremities of the visible Object; and the Rays belonging to each Pencil will be parallel to their respective *Axes*. Besides, the spurious Rays will be all cut off by the Plate M M; which makes the Vision very distinct. This Telescope is not only good for common Eyes, but the Rays that enter the Eye will be made to converge a little for the *Presbyta*, or to diverge a little for the *Myopes*, by means of a Skrew fixed to the Arm of the little Concave, to remove or to bring it forward upon Occasion. [N. B. *The Reason why Mr. HADLEY uses a double Eye-Glass instead of a single one proposed by Dr. GREGORY, is, to prevent the object being coloured at the Edges of the Aperture.*] The seventh Figure is drawn in the quarter Proportion of an Instrument of 12 Inches focal Length.

B B is the larger concave *Speculum*, its focal Length A G.

FF is the smaller concave *Speculum*, its focal Length ID.

The Breadth FF is about $\frac{1}{50}$ of an Inch wider than the Hole CC in the larger *Speculum*.

N the first Eye-Glass is plano-convex.

S the second Eye-Glass plano-convex likewise, or rather a *Meniscus*.

M a Plate with a small Hole in it to exclude all foreign Light, with an Hole in it $\frac{1}{8}$ of an Inch.

RR the limiting Circle or Diaphragm.

The Arrows are the several successive Images of any Object.

T A B L E I.

If AGbe=3Inches=12Inches=18Inches=27Inches

And the Charge is	$\left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} = 36 = 49 = 66$	$\left. \begin{array}{l} \text{That is, the} \\ \text{Power of} \\ \text{Magnifying:} \end{array} \right\}$
12 $\frac{2}{3}$ or 13		

	B B=0.	7=	2.	=	2.	7=	3. 7
	I D=0.	82=	2.32=	3.22=	4.22		
	F F=0.315=	0.56=	0. 7=	0.88			
	C C=0.295=	0.54=	0.68=	0.86			
Focal Length of	} N=1.	48=	3.27=	3.97=	4.91		
Focal Length of	} S=0.	7=	1. 3=	1.54=	1.85		
	A D=3.	96=	14.66=	21.69=	31. 9		
	A N=0.	5=	0. 7=	0.75=	0. 8		
	N S=1.	4=	2. 6=	3.08=	3. 7		
	S M=0.	45=	0.76=	0.88=	1. 0		
	R R=0.	2=	0.37=	0.44=	0.53		

N.B.

N. B. These Calculations are for the Day, where Objects are to be magnified but little in proportion to what the Heavenly Bodies may be at Night ; for which the following Table gives the Proportions.

T A B L E II.

If A G be = 12 Inches = 18 Inches = 27 Inches ;

And the Charge 70 = 95 = 128

B B must be	= 2 .	= 2 . 7	= 3 . 75
I D	= 1 . 74 .	= 2 . 36	= 3 . 22
F F	= 0 . 4 .	= 0 . 47	= 0 . 56
C C	= 0 . 38 .	= 0 . 45	= 0 . 54
Focus of N	= 2 . 29 .	= 2 . 79	= 3 . 47
Focus of S	= 0 . 87 .	= 1 . 03	= 1 . 25
A D	= 13 . 95 .	= 20 . 64	= 30 . 56
A N	= 0 . 7 .	= 0 . 75	= 0 . 8
N S	= 1 . 74 .	= 2 . 06	= 2 . 5
S M	= 0 . 47 .	= 0 . 56	= 0 . 7
R R	= 0 . 25 .	= 0 . 29	= 0 . 36

The Breadth in the Hole of the Plate M must be $\frac{1}{35}$ of an Inch.

N. B. The varying the Length of A N, that is the Distance of the first Eye-Glass behind the fore Surface of the *Speculum* B B, alters the other Proportions but little ; so that if the Thickness of the *Speculum*, or other Circumstances require it, there is no need to keep exactly to the Numbers here set down for it.

These

These Telescopes are much more convenient for Day-Objects, and more easily managed by Persons who are not used to Telescopes in general, than the *Newtonian* Telescope: But again, they are not so convenient for Celestial Objects, especially such as are at a considerable Altitude in the Heavens; for Celestial *Phænomena* may be viewed quite up to the Zenith with the same Conveniency as at the Horizon, with Sir ISAAC'S Telescope; which cannot be done with the *Gregorian*. Besides, in looking at Celestial Objects, we may see them only with an Eye-Glass, and by means of one single Image, in using the *Newtonian* Telescope; whereas the *Gregorian* has always two Images.

In short, when once Micrometers can be usefully applied to these two Sorts of Reflecting Telescopes (which Artists are now endeavouring to do) the long Dioptrical Telescopes will be quite out of use, and Astronomical Observations will be made as certainly, much cheaper, and with more Expedition.

N. B. The *two* foregoing *Tables* of *Calculation*, concerning the *Gregorian Telescope*, being only sent me, by Mr. HADLEY, for my private Assistance in making that Instrument; upon my asking Leave to publish it, he answered, That as he did not design
to

to make it public, he had not been correct, farther than the *second* or *third* Place of the *Decimals*; and therefore could not give it out for perfect. But as it is sufficient for Practice, I thought proper to give it here. And, at the Close of this *Appendix*, I have added Mr. HADLEY's Letter to me with his new Calculations for the *Reflecting Telescope*.

I should do Injustice to Mr. HADLEY, if I ended this *Appendix* without taking notice of his admirable new Invention of a *Reflecting Quadrant*, for taking Altitudes and Angular Distances more accurately than ever yet was done; and with so much Ease, that the Shake of a Ship does not influence the Correctness of the Observation, it being only required to find the Object. This is an Advantage which no Quadrant ever yet had. I do not here give the Description of it, because it is fully done in the *Philosophical Transactions* (N^o. 420.) and the *Quadrant* may be had at Mr. B. Sisson's, the Corner of *Beaufort-Buildings* in the *Strand*; where a Book is given with the *Quadrant*, which shews its Use in the plainest manner. As soon as the common Prejudice against new Things is worn off, and the Instrument is well known, I do not believe any Ship will go a long Voyage without one of these excellent *Quadrants*.



ORIGINALS

Referred to in the foregoing

APPENDIX.

NUMB. I.

Ex J. GREGORII Optica promota, P. 93.

— *Tertius autem genus aureum nulla habet incommoda, & omnes priorum generum proprietates habere potest, &c.*



THE third Sort, which from its Value we may very well call a *golden one*, has no Inconveniences, and may have all the Properties of the other Sorts; provided the *Lens's* and *Specula* be rightly disposed; that is, if the last Image and the last but one be produced by *Specula*, and the rest by *Lens's*. We shall therefore here, for Example, describe a Telescope of this most perfect Kind. Let ADCE (*Pl. IV. Fig. 1.*) be a parabolical concave *Speculum*
K k
most

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 most exquisitely polished; *in whose Focus* *
 C is placed a small elliptic concave *Specu-
 lum*, having a common *Focus* and common
Axis with the concave parabolic *Speculum*,
 and let it be fixed in that Situation. Now
 the said *Focus* of that elliptic *Speculum*
 must be very near to its *Vertex*, and the
 other *Focus* of it must be very far from the
 same at F, in the common *Axis* produced
 beyond the parabolic *Speculum*; and thro'
 the *Vertex* of the parabolic *Speculum* must
 be made a round Hole M N, in which Hole
 must be placed a Tube having the same *Axis*
 with the *Specula*, and big enough to re-
 ceive the Rays of a Visible reflected from
 the elliptic concave *Speculum*; and let it
 be continued to L very near to F; and at
 L let a *Lens* of Crystal, convex towards
 the *Specula*, with the Convexity of a Co-
 noid, and the Density of the Crystalline (of
 the Eye) be fixed, whose exterior *Focus*
 must be at F, and which must be plane to-
 wards the Eye, and likewise have the same
 common *Axis* as the *Specula* and the Tube.
 This will be the Way to make an excellent
 Telescope for *Presbytæ*: For distant Objects
 seen through the Tube will appear very
 distinctly, magnified very near in the *Ratio*
 of the Distances of the *Vertices* from the

* The Author means, *Near whose Focus.*

common *Foci*; and enlightened in the same manner as a visible would be, when seen under such an Angle; provided the Diameter of what produces the last Image be big enough to suffer the *Uvea* of the Eye to be filled with the Rays: And how that may be done, we have taught in the *Scholium* of the 51st of this Book, &c.

N U M B. II.

Copy of a Letter written by Mr. NEWTON to a Friend of his (taken from Mr. Collins's Transcript.)

*Trin. Coll. Cambridge,
Feb. 23, 166 $\frac{1}{2}$.*

I Promised in a Letter to Mr. *Ent* to give you an Account of my Success in a small Attempt I had then in hand; and it is this: Being persuaded of a certain Way whereby the practical Part of Optics might be promoted, I thought it best to proceed by degrees, and make a small Prospective first, to try whether my Conjecture would hold good or not. The Instrument that I made is but six Inches in Length; it bears something more than an Inch Aperture, and a plano-convex Eye-Glass, whose Depth is $\frac{1}{2}$ or $\frac{1}{7}$ Part of an Inch: So that it magnifies

about 40 times in Diameter ; which is more than any six Foot Tube can do, I believe, with Distinctness. But by reason of bad Materials, and for want of good Polish, it represents not Things so distinct as a six Foot Tube will do ; yet I think it will discover as much as any three or four Foot Tube, especially if the Objects be luminous. I have seen with it *Jupiter* distinctly round and his Satellites, and *Venus* horned. Thus, Sir, I have given you a short Account of this small Instrument ; which though in itself contemptible, may yet be looked upon as an Epitome of what may be done according to this Way : For I doubt not but in time a six Foot Tube may be made after this Method, which will perform as much as any 60 or 100 Foot Tube made after the common Way ; whereas I am persuaded, that, were a Tube made after the common Way, of the purest Glass exquisitely polished with the best Figure that any Geometrician (*Des-Cartes*, &c.) hath or can design ; (which I believe is all that Men have hitherto attempted or wished for) yet such a Tube would scarce perform as much more as an ordinary good Tube of the same Length. And this however it may seem a paradoxical Assertion, yet it is the necessary Consequence of some Experiments which I have made concerning the Nature of Light, &c.

N U M B. III.

Copy of Part of a Letter to Mr. JOHN COLLINS.

Aberdeen, 6 August, 1672.

— IT is like indeed that Mr. NEWTON his Telescope may have an Advantage above that which I mention'd in my *Optica promota*, because the Eye-Glasse is so near the plane Mirroir; yet the Obliquitie of the Mirroir hindereth someqt: Nevertheless my Telescope hath one Advantage also verie considerable; for the same concave Mirroir together with the same plane-convex Eye-Glasse may give the same Object-Mirroir any desired Charge, &c.

*Your humble and
obliged Servant,*

J. GREGORIE.

N U M B. IV.

St. Andrews, 26 Sept. 1672.

S I R,

Yours of the third of *August* I received a considerable time ago, &c. — I have cast an Eye on Mr CASSEGRAIN his Telescope,

scope, which seemeth to be the same with that in my *Optica promota*, Page 94. onlie he hath a convex Speculum F (*Plate IV. Fig. 5.*) in place of my concave; which is no great Alteration. I think myself therfor obliged to answer to these Disadvantages Mr. NEWTON finds in it. I mak therfor F an plane Speculum, and then almost the whol Disadvantages evanish, except onlie the third; and for that, ther is an Advantage as considerable, if not more; *viz.* that the Distance EF groweth almost the one halfe lesse; and therfor the Errors of the concave CD ar also diminished upon the plane F by one halfe. Ther is yet another Advantage of this Telescope, that it will be little more then halfe the Lenth of Mr. NEWTON's, and doe the same Effect. Nevertheless of these Disadvantages which Mr. NEWTON mentioned, even with a concave or convex Speculum, this Telescope may be worth the trying; seing the Eye-Glasse and Speculum F being moveable, the Speculum CD can have by their Help anie desirable Charge; which I think a great Advantage. What I ether did or said needeth not discourage anie; for I speak ther onlie of the hyperbolick and elliptick Glasses and Speculum, which wer attempted in vaine, as it is clear from the Sense of the Word. As for my Experiment with Mr.

Rives,

Rives, he could not polish the large Concave upon the Tool: And I (not knowing anie Advantage of the Catoptrick Telescope above the Dioptrick, save onlie the Shortnes and Similitude betwixt the Circle and Parabola, which is greater than that betwixt the Circle and Hyperbola) imagined that this great Defect in the Figure wold easilie counterbalance these two pettie Advantages. Upon this Account, and being about to go abroad, I thought it not worth the Pains to truble my self anie further with it; so that the Tube was never made: Yet I made some Tryals both with a litle concave and convex Speculum; which wer but rude, seing I had but transient Views of the Object; being so possessed with the Fancie of the defective Figure, that I wold not be at the Pains to fix everie thing in its due Distance. Ther is no such Exactnes required in the Speculum F as in the Speculum CD; but indeed more than in the Eye-Glasse. I suppose ther is no Question that direct Rayes have the Advantage of oblique; seing a Ball thrown directlie on a rough Wall hath a more regular Reflection then when it is thrown obliquelie. However this is not derogat from Mr. NEWTON, whose Discoveries hath made the Catoptrick Telescopes preferable to the Dioptrick, &c.

This Telescope with the plane Speculum will indeed lose maire of the best Raies; but
these

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these nevertheless are always less than $\frac{1}{4}$ of
the whole, the Eye-Glasse being advanta-
giouſſie ſituat; which Defect ſome perchance
may think recompens'd by the Shortnes of
the Telescope.

I ſuppoſe ther is no great Hazard of over-
charging the Telescope by the concave or
convex Speculum; for the Charge can be
changed at Pleaſur: Nether is it probable
to me that the Errors of the Object-Specu-
lum are made more ſenſible (the magnifica-
tion being alwaies the ſame) by a concave
or convex Speculum and an Eye-Glaſſe,
then by an plane Speculum and an Eye-
Glaſſe, ſave onlie upon the Account of
greater Diſtances; which I think the onlie
Defect of this Telescope.

Your humble Servant,

J. GREGORIE.

N U M B. V.

Cambridge, Jan 6, 167 $\frac{1}{2}$.

S I R,

AT the reading of your Letter I was
ſurprised to ſee ſo much Care taken
about ſecuring an Invention to me of which
I have hitherto had ſo little Value. And
there-

therefore since the *Royal Society* is pleased to think it worth the patronizing, I must acknowledg it deserves much more of them for that; than of me, who, had not the communication of it been desired, might have let it still remained in private, as it hath already done some Yeares.

— The Description of the Instrument you sent me is very well, only the radius of the concave metal, which you put 14 Inches, is more justly $12\frac{2}{3}$ or 13 Inches; and the radius of the eye-glass, which you put half an inch, is the twelfth part of it, if not less: For the metal collects the sun's rays at $6\frac{1}{3}$ inches distance, and the eye-glass at less than $\frac{1}{6}$ part of an Inch Distance from its vertex. By the tools also to which they were ground I know their Dimensions; and particularly measuring the Diameter of the hemispherical Concave, in which the Eye-Glass was ground, I find it the sixth part of an Inch.

Perhaps it may give some Satisfaction to Mounſieur HUYGENS, to understand in what Degree it represents things distinct and free from colours; and to know the aperture by which it admits light: And after the words [— *Versus Focum E reflectatur.*] (*Pl. IV. Fig. 2.*) it may not be amiss to add this note.

Conferendo distantias foci istius a verticibus Lentis & speculi concavi, hoc est, EF $\frac{r}{6}$ & ETU $6\frac{1}{3}$ dig. prodit ratio 1 ad 38; qua indicatur objecta 38 vicibus circiter ampliari.

And to this proportion is very consentaneous the observation of the crowns on the weathercock: For the scheme represents it bigger by $2\frac{1}{2}$ times when seene through this then when through an ordinary perspective. And so supposing that to magnify 13 or 24 times, as by the Description it should, this by the experiment proportionably must magnify almost as much as I have assigned it.

To the objection, that with it objects are difficultly found, I may answer, that that's the inconvenience of all Tubes that magnify much; and that after a little Use the Inconvenience will grow less: For I could readily enough find any day-objects, by knowing which way they were posited from other objects that I accidentally saw in it; but in the night to find Starrs, I confess is troublesome enough. Yet this may be easily remidied, by two sights affixed to the iron Rod by which the tube is susteined. And such I once intended should have beene made before I sent it away from mee, but that I thought the defect would not be adjudged

adjudged material. If such sights be not found a sufficient remedy, there may bee an ordinary perspective glass fastened to the same Frame with the tube, and directed to the same object; as DES-CARTES in his Dioptrics hath described for remedying the same Inconvenience of his best telescopes.

The plane side of the eye-glass is apt to bee soiled with dust falling upon it; and therefore the little leaden Ring put into the orifice of the bigger leaden barrel to moderate its aperture, must be sometimes taken out, and the glass wiped with leather done upon the small end of a stick, or other such like contrivance; but care must bee taken that the said Ring bee not lost, for without it Objects appear very confused at the edges of the apparent space. So if the concave metal contract any dullness by moisture, or otherwise, it ought to be taken out and rubbed with gentle leather, but not with Putty, or any thing that may weare the metal.

I am very sensible of the Honour done me by the Bishop of *Sarum*,* in proposing mee Candidate; and which, I hope, will bee further conferred upon me by my Election into the Society. And if so, I shall endeavour to testify my gratitude, by communicating what my poor and solitary endeavours

L 1 2

can

* Seth Ward, D. D.

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can effect towards the promoting your Philosophical designs.

S I R,

I am

*Your very humble
Servant,*

I. NEWTON.

N U M B. VI.

St. Andrews, 7 March, 1673.

S I R,

I Have received yours, dated *Febr. 20*, together with Mr. NEWTON's Answer, with which I am exceedingly satisfied. I am much engaged to you both for the Pains ye have been at. I am almost convinced that oblique reflection causeth more Light then the direct; but I am not fully perswaded that it is more regular. I conceive that the rudelie polished plate of metall in an oblique position causeth the image appear more distinct, because the obliquitie hideth the concavities, so that no rayes come to the eyes but from the tops of the litle *tubercula*; which ar certeinlie best polished, the other rayes which confused
the

the image being kept away. But if the plate be exactlie polished (I speak here as to sense) the position must be so oblique, before the insensible concavities can be hid, that the plane shall almost turn, to the sight, in a line. I Grant that I have been mistaken in that first Advantage which I mentioned: for the plane Speculum F having certenlie (as all human artifice hath) some errors in it, causeth greater prejudice by their being remote from the focus than being near to it; and in it ther is none at all caused: wher if it could be placed, and an near and direct aspect had of it, this wer certeinlie the best telescope of this Sort.

It is true indeed, that in telescopes with convex or concave *Specula* to double the charge, the lenth must be almost doubled; but to double is a great alteration, and hardlie sufferable (as I suppose) in werie good glasses, if the least charge be considerable. But I understand not how the charge can be altered at all with the same glasses in Mr. NEWTON's Telescope; for I know nothing of that which was described to Mr. OLDENBURGH. It is true that eyeglasses can be charged in all Telescopes if they be at hand of the required depth. I think there is no great Hazard in these telescopes of overcharging, seing the charge of the Eye-Glasse can be diminished at Pleasure;

fur; nether upon this account needs the angle of vision be so small, seing it is equal to the angle of the eye-glasse from its focus, its other focus being the litle speculum; nor the darknes at all augmented, if the apertures of the speculums be proportional to the diameters of the Spheres. But above all things I desire to know this; that seing the Image made by the great Speculum may be esteemed a smal visibile, and seing Mr. NEWTON in the *Transactions*, Page 3080, thinketh it fitter to mak an microscope or tube to behold an smal visibile of one concave Speculum and one eye-glasse, rather then with one single Eye-Glasse, and much rather than with one plane Speculum and with one Eye-Glasse: wherfor also to look to this smal visibile, the first also fould not be preferred to the last. This image indeed is not capable of such magnification as an visibile is; yet I am hardlie sensible how this fould cast the ballance, taking in the defects of a plane Speculum, togither with other inconveniencies in taking up the object. [I said indeed, that hyperbolick and elliptick Glasses wer tryed in vaine; but I spoke not so of spherick Speculums (as Mr. NEWTON's Words seem to imply, *Transactions*, Page 4059.) for any thing I did deserves not the Name of a Tryall, seing Mr. Rive and Mr.

Mr. Cox both know that the great Speculum was polished onlie with a Cloath and puttie: nether thought I it worth the pains at that time to be serious about further enquiry in that busines; for they undertook indeed to polish a less Speculum to me upon the tool. I am not yet fullie convinced which of these two ways have the advantage; albeit I incline more to Mr. NEWTON's, especialie because of the smal distance betwixt the plane Speculum's focus and the eye. However, Experience must determine all; neither am I concerned how it happen. I had no Intention that my thoughts of these telescopes should be printed; my designe was onlie befor, as now, that (if ye thought fitt, otherways not) ye might send them to Mr. NEWTON, &c. —

Mr. NEWTON's Discourse of Reflection puts me in mind of a Notion I had of burning glasses several years ago; which appears to me more usefull than subtile. If ther be a concave Speculum of glasse, the leaded convex Surface having the same Center with the concave (or to speak preciselie, albeit perchance to little more Purpose, let the radius of the convexitie be c , the thicknes of the glasse *in axis transitu* f , the Radius of the Convexitie equal to $\frac{9c^2 + 18cf + 5f^2}{9c + 5f}$) this Speculum shal have
the

the *Foci* of both the Surfaces in the same Point; and not onlie that, but all the Rays, which ar reflected betwixt the two Surfaces, fal in their Egresse come *quam proximè* to the common Focus. The making of such an Speculum requireth not much more Airt then an ordinar plane Glasse, seing great Subtiltie is not necessar here: So that I believe they, who mak the plane Miroir-Glasses, wold mak one of these three Foot in Diameter for four or five L. ft. or litle more: For I have seen plane Glasses almost of that Bignes sold even here for less Money. Now seing (as Mr. NEWTON observeth) that al reflecting Metalls lose more then $\frac{1}{3}$ of the Rayes: This concave Glasse even *ceteris paribus*, wold have an great Advantage of a Metall one; for certainlie an exactlie polished thin Miroir-Glasse of good transparent Mater, after a few Reflections, doeth not lose $\frac{1}{4}$ of the Rayes: And upon other Accounts this hath incomparable Advantages, seing it is more portable, free from tarnishing, and above al hardlie $\frac{1}{2}$ of the Value. The great Usefulness of Burning Concaves, this being so obvious, and as yet (for qt. I know) untouched by anie, makes me jealous that there may be in the Practise some Fallacie. Ye may communicate this to intelligent Persons, and especially to Mr. NEWTON; assuring

assuring him, that none hath a greater Veneration for him, admiring more his great and subtile Inventions, then his and

Your humble Servant,

J. GREGORIE.

If ye please, let me hear with the first convenience what may be judged the result of this burning concave: for I am as much concerned, to be undeceived, if ther be any insuperable difficultie, as to be informed of an most surprizing success. I have spokē of it to severals here, but al wer as ignorant of it as my self, &c.

[I desire yet to be more particular in the mater of telescopes. I suppose an 4 foot telescope have the aperture 6 inches; the litle concave having the aperture $\frac{3}{4}$ inches, may magnifie 8 times, the radius being 1 foot. In this case the hole in the midle of the great concave is onlie $\frac{1}{4}$ inche, which being fitted with an eye-glasse equallie convex on both sides, amplifying the Charge of the litle concave 24 times, doeth mak an telescope magnifying the object 190 times (which is no extraordinar charge, seing Mr. NEWTON's table giveth 171, and might be much less without inconvenience) taking in an angle of vision of

M m

above

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above 20 Degr. and with this ther is not
lost $\frac{1}{6}$ of the rays. with the Losse of
 $\frac{1}{36}$ of the Rayes it might magnifie not above
144 times, and tak in an angle of vision
of above 28 Deg. with al this the midle of
the object is illustrat with all the rays
which the aperture of the great concave
doth reflect. by these means I think that
I keep of from these two inconveniences
mentioned by Mr. NEWTON in the seventh
particular of his considerations. the e-
vent of these other considerations, as I sup-
pose, can onlie be determined exactlie by
experience.]

N U M B. VII.

S I R,

HAVING perused Mr. GREGORY's candid
reply, I have thought good to send
you these further considerations upon the
differences that still are between us. And
first, that a well polished plate reflects at
the obliquity of 45 degrees more truly
than direct ones, seems to me very certain:
for the flat *tubercula*, or shallow valleys,
such as may be the remains of scratches al-
most worn out, will cause the least errors
in the obliquest rays which fall on all sides
the

the hill, excepting on the middle of the foreſide and backſide of it; that is, where the hill inclines directly towards or directly from the ray: for if the ray fall on that ſection of the hill, its error is in all obliquities juſt double to the hill's declivity: but if it fall on any other part of the hill, its error is leſs than double, if it be an oblique ray, and that ſo much the leſs, by how much the ray is obliquier; but if it be a direct ray, its error is juſt double to the declivity, and therefore greater in that caſe. I preſume Mr. GREGORY, if you think it convenient to tranſmit this to him, will eaſily apprehend me.

How the charge may be varied at Pleaſure in my teleſcope, will appear by this Figure; where A repreſents the great concave, E the Eye-Glaſs, and B C D a Priſm of Glaſs or *[Plate 4, Fig. 8.]* Cryſtal, whoſe ſides B C and B D are not flat, but ſpherically convex; ſo that the rays which come from G, the focus of the great concave A, may, by the refraction of the firſt ſide B C, be reduced into paralleliſm, and, after reflexion from the baſe C D, be made by the refraction of the next ſide B D, to converge to the focus of the eye-glaſs H.

The Teleſcope being thus formed, it appears how the charge may be altered, by

276 LETTERS between Dr. GREGORY
varying the distances of the glasses and
speculum.

As for the Objection, That Mr. GREGORY's Telescope will be either overcharged, or have too small an angle of vision, &c. I apprehend that the difference between us lies in limiting the Aperture of the eye-glass. Mr. GREGORY puts it equal to that of the little concave; but I should rather determine it by this proportion; That if a middle point be taken between the Eye-glass and its focus, the apertures of the eye-glass and concave will be proportional to their Distances from that Point: That is,
Plate 4, Fig. 10.] suppose A B the little concave, E F the eye-glass, G H their common focus or image, and K their mean distance between G H and E F; from the extremities of A B draw A K and B K, butting on the Eye-Glass at F and E, and E F shall be its Aperture. The reason of this limitation is, that the superfluous light which comes on all sides of the speculum A B to the space G H, in which the picture of the object is made, may fall besides the eye-glass: For if it should pass through it to the eye, it would exceedingly blend those parts of the picture with which 'tis mixed; and such are those parts of it which extend themselves beyond the lines A K, B K. As I remember, I said in my
former

former letter, that the scattering Light which falls on the eye-glass will disturb the vision; and this is to be understood of any straggling light which comes not from the picture; but if it come from the picture to the eye-glass, the disturbance will be much greater, so as not to be allowed of. Against the first, I see no very convenient remedy; and against the last, none but assigning a small Aperture to the eye-glass; supposing the Telescope is used in the Day-time, or in twilight, or to view the Moon, or any starr very neare her, or neare the brighter Planets. And if for this reason the Aperture be limited by any rule, the angle of vision will become very small, as I affirmed: For Instance, in that case where Mr. GREGORY in his Postscript puts it above 20 degrees, it will be reduced to less than half a degree. Yet I confess there is a way by which the angle of vision may be something enlarged; but it will not be very considerable, unless the eye-glass be also deeper charged.

Why I assign a concave with an eye-glass to magnify small Objects (in *Transactions*, Page 3080.) and yet an eye-glass without such a concave to magnify the image of the great concave, which is equivalent to a small object, is, because that image doth not require to be magnified so much

much as an object by a Microscope ; and further, because the angle of the penicil of rays which flow from any point of the small object, that the object may appear sufficiently luminous, ought to be as great as possible ; and a concave will with equal distinctness reflect the rays with a greater angle of the penicill than a Lens ; but in the Telescope the Angles of those Pencils are not so great as to transcend the limits at which an eye-glass may with sufficient distinctness refract them : And therefore in these instruments I chose to lay all the stress of magnifying upon the eye-glasses. In Microscopes also I would lay as much stress of magnifying upon the eye-glass as it is well capable of, and the excess only upon the Concave.

Concerning my citation of Mr. GREGORY against Mons. CASSEGRAIN, the force of it lies only in the inference that Optic Instruments most probably, according to M. CASSEGRAIN's design, have been tried by Reflexion ; which I think I might well infer without having regard to the specific figure of the speculum which Mr. GREGORY there spake of : And therefore I think it cannot be said that I made him speak of spheric figures, where his meaning was of hyperbolic and elliptic ones. But if I should be so understood, because I put the figure of the
great

great concave to be spher-
 ical, I know not what
 of Consequence makes
 For it is not probable
 attempt Hyperbolic
 Speculums, until the
 ones had been first tri-

ed-
 way
 on :
 ould
 gures of
 spherically

And accordingly the trial of Mr. GREGORY with Mr. *Reize* was by a spherical Figure: Which trial, although I am now satisfied that it was made very rudely, yet by the Information which I had of it when I wrote the letter about Mr. CASSEGRAIN's design, I apprehended it to have been made with very great diligence and curiosity, as I signified in my former letter at large. And this I hope may excuse me for speaking of it in the *Transactions* as if it had been tried with more accuracy than really it was. And thus much concerning the Telescope.

The design of the burning Speculum appears to me very plausible, and worthy of being put in practice. What Artists may think of it I know not; but the greatest difficulty in the practice that occurs to me is, to proportion the two surfaces so, that the force of both may be in the same point according to the Theory. But perhaps it is not necessary to be so curious; for it seems to me that the effect would scarce be sensibly

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tween Dr. GREGORY

sides should be ground
age of the same tool,

ur humble Servant,

J. NEWTON.

N U M B. VIII.

St. Andrews, 13 May, 1673.

S I R,

I Received lately your's, dated the 19th April, together with Mr. NEWTON's to you; for whose faire correspondence I give you Both hearty Thankes: To which I have onlie these few things to say. As to his first, I understand not well his meaning: an oblique position seemeth to expose al its inequalities more fullie to the rayes; and ether altogither to hide the lowest of the regular Surfaces, or otherways to reflect the Rayes coming from them on the adjacent *tubercula*.

His way of varying the charge is indeed exceedingly ingenious; but I think those Surfaces too lyable to the errors of the artificer's Hand. The opacitie of the glasse prisme, together also with the irregularitie which

which he hath discovered in Refraction, may help to darken and confufe the Sight.

As for the next, I know not if it be wor-
thie of the paines to look with excellent
Telescopes on terrestrial bodies: For as the
Object is magnified, so is the grossenes of
our Atmospher to our sense encreased; so
that the one hindereth as the other helpeth.
In celestial Observations any little thing ap-
plied to one or more sides of the litle Spe-
culum, may stape the Rayes of the Moon,
or anie other of the brighter Planets, if
these be also thought worthie noticing. I
suppose that al these adventitious Rayes
may be hindered even in Day-light, by put-
ting in the Focus of the eye-glasse towards
the eye an thin Plate of some Metall with
an litle round Hole in the midle in Diameter
 $\frac{1}{12}$, $\frac{1}{20}$, or $\frac{1}{30}$ of an Inch; which is calcu-
late so, as the Distance of the Eye-Glasse
from the litle Concave is to the Distance of
the Eye-Glasse from its Focus, so is the
Aperture of the litle Concave to the Dia-
meter of this Hole. It is true, at some
times this may hinder some of the Rayes,
but they ar always the worst; and by en-
creasing the Aperture of the litle Concave
not much above what my methode requirs,
it will hinder non at all. I could not have
judged that Mr. NEWTON had thought on
this Inconvenience in Mr. CASSEGRAINE his

Telescope, seing it seemeth to me, even in his own Microscope, *Transactions*, Page 2080. for not onlie the direct Rayes of the Object O (nevertheles that it be looked to onlie with Day-light) but also these proceeding from the Objects befor the Concave ar always scattered through the whol Image; nether doe I see how it can be exactlie helped.

That we may se what Effect this scattered Light may cause in the Sight, let us suppose the Telescope to magnify 160 times, and the Aperture of the great Concave to contain 8 times the Aperture of the Eye-Glasse or litle Concave, and the Object to be a Planet apparent Diameter $\frac{1}{2}$ of a Minute, in whose Image ther passeth the Rayes of another Planet of the same apparent Bignes and Brightnes. The Angle of Vision is about 16 Deg. the Planet appears in an Angle of $1\frac{1}{3}$ Deg. that is to say, it illustrats so much of the *Retina*. Now the other Planet illustrats (I take no Notice of the litle Concave, which is to my Disadvantage, seing it keepeth of manie of these Rayes) 16 Deg. of the *Retina*. Now because of the Aperturs, ther ar 64 times as manie Rayes in $1\frac{1}{3}$ of a Deg. as in 16; that is to say, these adventitious Rayes have but $\frac{1}{9216}$ of the Splendour of the Image; which I think hardlie sensible. The Brightnes indeed of the Moon wer near $\frac{1}{2}$ and

$\frac{1}{2}$ and not sufferable ; which therfor is to be helped be some of the foresaid Means. In the Twylight the Inconveniencie may be for the most part verie inconsiderable, and perchance somtimes (as also other adventitious Rayes) advantagious by making insensible the Circumradiance of celestial Bodies. All this is supposing the Eye-Glasse convex ; for if it be concave, the Effect is otherways.

As to his last, I imagine that all Images doe require (*cæteris paribus*) to be magnified as much as may be. Nether doeth his other Reason appear to me ; for Penicills of the same Angles ar more trulie reflected by an Concave, then refracted by a Lens. And albeit in Telescopes the said Angle transcend not the Limite of a Lens commonlie assigned, yet surelie the more it is exceeded by this Limite, it is so much the better. And al this is observed in my Designe ; yea ther is 3 times as much Strefs of Magnifying also laid upon the Eye-Glasse as on the litle Concave. It may also be noticed, that here ther ar no verie smal Sizes of Spheres to be polished ; which can hardlie be done (as I suppose) to Precifenes. It is possible that even in Telescopes ther may be more strefs laid on the Eye-Glass then it can carrie ; especialie in the extreme Penicills, wher the Incidence is oblique, and Refraction perhaps so great, that $\frac{1}{25}$ of it may be sensible.

sensible. Also an ordinary Microscope suffers no Aperture above the Limite of a Lens; and nevertheless it doeth much more then one simple Lens, or else the Worlde hath been exceedingly deceived. I dare not confidentlie affirme, that ordinary Microscopes may outdoe any Improvment of one Lens; but if they doe, I think it more then an probable Argument, that my Project shall exceed Mr. NEWTON's; seing beside the onlie Disadvantage which I see in mien (to wit, the Distance of the Glasses) it hath the great Irregularitie of Refraction.

I think nothing can be inferred concerning the Tryal of my Telescope from my Assertion, seing the Tryal was after that Assertion; but Mr. NEWTON could not be supposed to know this.

S I R,

Your most humbe Servant,

J. GREGORIE.

F I N I S.

and Mr. NEWTON. 285

TO

The Reverend Dr. DESAGULIERS.

S I R,

I Have inclosed your Papers, (which were left with me last Week;) what I have added, you will be pleased to make use of, or not, as you think fit. The first is a Rule for the Parts of the *Gregorian* Telescope, of the same Nature with what I formerly delivered to Mr. *Molineux*, relating to that of Mr. *Cassegrain*. I have carried the Tables for those with two Eye-glasses, both for Night and Day, as far as it seems probable to be of any Use. I am,

S I R,

Your most humble Servant,

Dec. 9th,
1734.

J. HADLEY.

P. S. I have just received the Favour of yours. I imagined, the Plate with the small Hole to exclude foreign Light in Telescopes, to have been *my own first thought*, but find Mr. *Ja. Gregory* had had the *same* before.

O o

The

*The Proportions for the several parts
of a Catadioptric Telescope of the Form
proposed by Mr. JAMES GREGORY.
PLATE 4. Fig. 8.*

LET, A D represent the common Axis of the Telescope, and 2 concave Specula B B and F F. Suppose A G the focal length of the Speculum B B, whose proper Aperture B B, and Charge, are likewise known. Let C C be the Breadth of the Perforation. F F the Breadth of the smaller Speculum equal to, or a little greater than C C. I its Focus; N the Eye-glass, N A its focal Length, and M a Plate, with a small hole to exclude all foreign Light. And let it be required to take in at one View so much of the Object as may appear through the Telescope under a given Angle, *viz.* = C N C. To do this with the loss of the fewest Rays of Light near the Axis, the Proportions should be as follow.

Call A G *a.*

B B *b.*

The Power or Charge *m.*

The Ratio of twice the semitangent of the apparent Angle of Comprehension required C N C, to Radius *i. e.* $\frac{C C}{A N} = n.$

Then H H the Breadth of the Image of so much of the Object as is seen at once, will be = $\frac{n}{m} a.$

[N. B. If instead of $\frac{n}{m} a$, you substitute c for HH , the Algebraic Expressions become something more simple, for which reason I have added them.]

The Breadth of the Perforation CC ; of the great Speculum $= \frac{na + \sqrt{na} \times \sqrt{na + mb}}{m}$,

or $c + \sqrt{bc + cc}$.

The focal Length of the small Concave ID .

$$\frac{a \times \sqrt{na} \times \sqrt{na + \sqrt{na + mb}}}{na + mb + 2 \sqrt{na} \times \sqrt{na + mb}}$$

or $\frac{a \times c + \sqrt{bc + cc}}{b + c + 2 \sqrt{bc + cc}}$.

The Distances of the Specula, *i. e.* AD . $a + \frac{a \sqrt{na}}{\sqrt{na + mb}}$, or $a + \frac{a \sqrt{c}}{\sqrt{b + c}}$.

The focal Length of the Eye-glass, and its Distance from A , *i. e.* AN .

$$\frac{a}{m} + \frac{\sqrt{naa + mab}}{m \sqrt{n}}, \text{ or } \frac{c + \sqrt{bc + cc}}{n}.$$

The Distance of the Plate M , behind the Eye-glass NM , $= \frac{DN \times AN}{DA}$.

The Breadth of the Hole in $M = \frac{NM \times CC}{DN}$.

If a double Eye-glass be used with this *Telescope*, to prevent the Objects being coloured

loured near the Edges of the *Area*, the Image of the Object must be thrown back by the smaller Concave, so far behind the great *Speculum*, that there may be room enough to place the first Eye-glass N at a sufficient Distance before it, and the Algebraic Expressions of the several Parts become much more complex, wherefore I have omitted them, and added the Proportions for the following Sizes.

For the Night.

If A G be 40 inches,		60	90
and Power 172.		234.	317.
BB	4.9	6. ^{$\frac{2}{3}$}	9.0
ID	4.28	5.88	8.01
FF	0.67	0.81	0.97
CC	0.66	0.80	0.96
focus N	4.23	5.15	6.29
length S	1.52	1.82	2.2
AD	44.72	66.4	98.68
AN	0.9	1.1	1. ^{$\frac{1}{8}$}
NS	3.04	3.64	4.4
SM	0.8	0.93	1.13
RR	0.43	0.52	0.63

For the Day.

If A G be 40 inches, and the Power 86.			
BB =	4.9	AD =	6.74
ID =	5.95	AN =	0.9
FF =	1.0	NS =	4.44
CC =	0.99	SM =	1.15
N =	6.02	RR =	0.63
S =	2.22		

F I N I S

Errata & Corrigenda.

Pag.	Line	For	Read
11	23	A B C	p B c
60	2	$-2dx \times 2xx \text{ I} \times 2xx$	$-2dx + 2xx \text{ I} \times 2xx$
93	22	$\overline{+6arz} + 2cvz \overline{+4acr}$	$\overline{+6arz} + 2crz \overline{+4acr}$
ibid.	27	$\frac{2ab + 2ar}{a + b}$	$\frac{2ab + 2ar}{a + b}$
117	2	after collected, insert,	or diverge as if they were collected.
ibid.	7	dele from, whence, to therein,	
		dr	$d x$
124	19	$x = \frac{\quad}{2d + r}$	$r = \frac{\quad}{d - x}$
128	4	Concave $b - cr$	Convex. $br - cr$
131	19	$d = \frac{\quad}{2 c}$	$d = \frac{\quad}{2 c}$
133	6	$:: 2 d + rr ;$	$:: 2 d + r : r ;$
ibid.	7	$:: r = 2d . r .$	$:: r - 2 d : r .$
139	20.	$+ \frac{1}{2} r s t$	$+ \frac{4}{3} r s t$
140	17	after is, insert	$\frac{6 . drr}{6 . dr}$, or.
143	ult.	$+ I + s R t$	$I s + R t$.
146	13	or	and.
147	24	$4rst + 64py - 2rt y$.	$4rst + 6rsy - 2rt y$.
149	8	after have insert	$d = \frac{-6rrr}{6rr - 6rr}$ or.
151	3	$\frac{I}{I - R}$	$\frac{R}{I - R}$
164	9	refracted,	reflected,
165	14	Bf _n , the second D _φ x,	Bf _u , the second D _φ x,
166	3	$6a + 2c - r$	$6a + 2c - Z$
168	10	$\frac{\frac{15}{288}}$	$\frac{\frac{15}{348}}$
ibid.	18	$I \frac{\frac{3}{48}}{\frac{1}{48}} a$	$I \frac{\frac{3}{48}}{\frac{1}{48}} a$
ibid.	19	$\frac{1}{25} a$	$\frac{1}{11} a$
170	3	after PROP. 1. refer to	Plate III.
180	1	after PROP. 2. refer to	Plate III.
183	9	after Microscopes, refer to	Plate III.
187	2	after PROP. 3. refer to	Plate III.
194	8	after PROP. 4. refer to	Plate III.
196	15	E L	D L
197	23	after PROP. 5. refer to	Plate III.

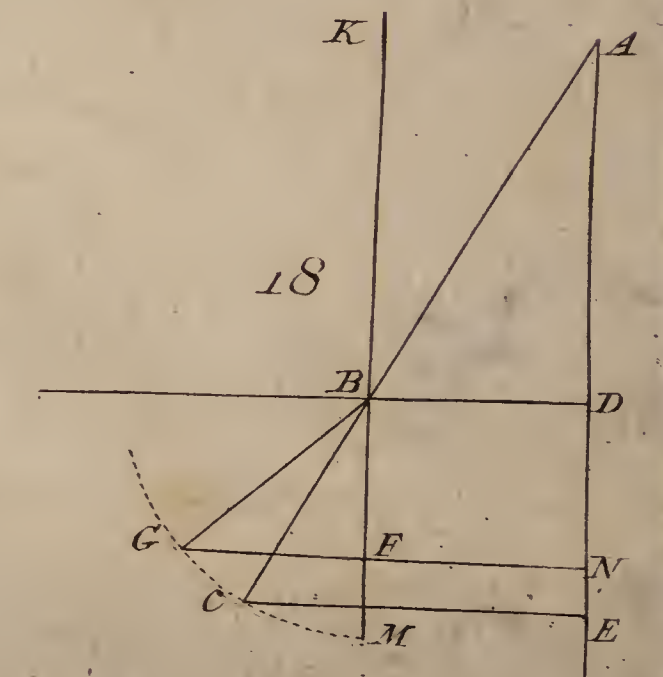
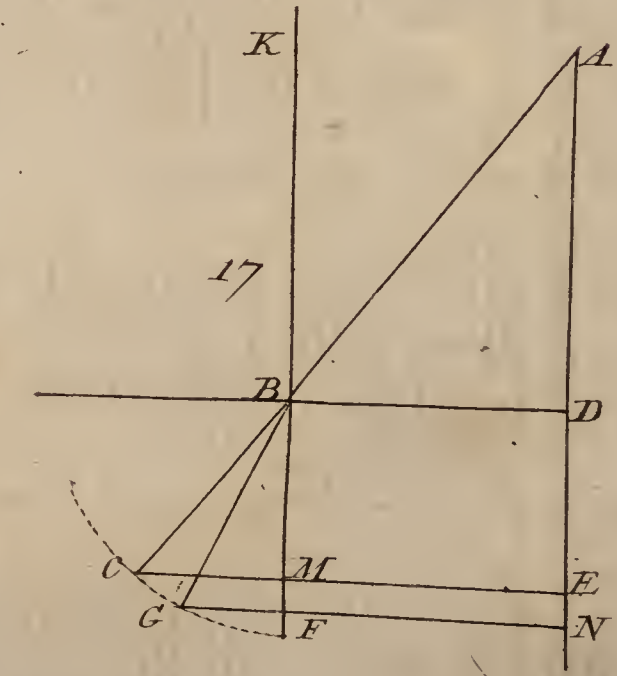
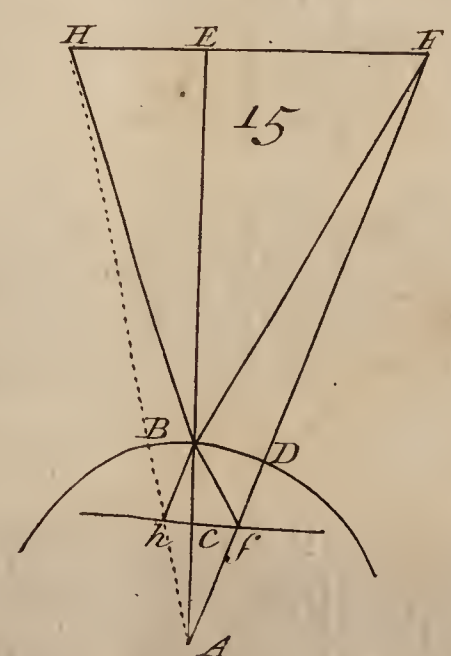
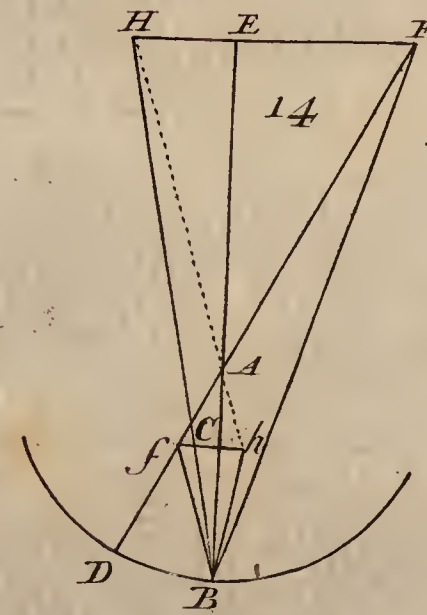
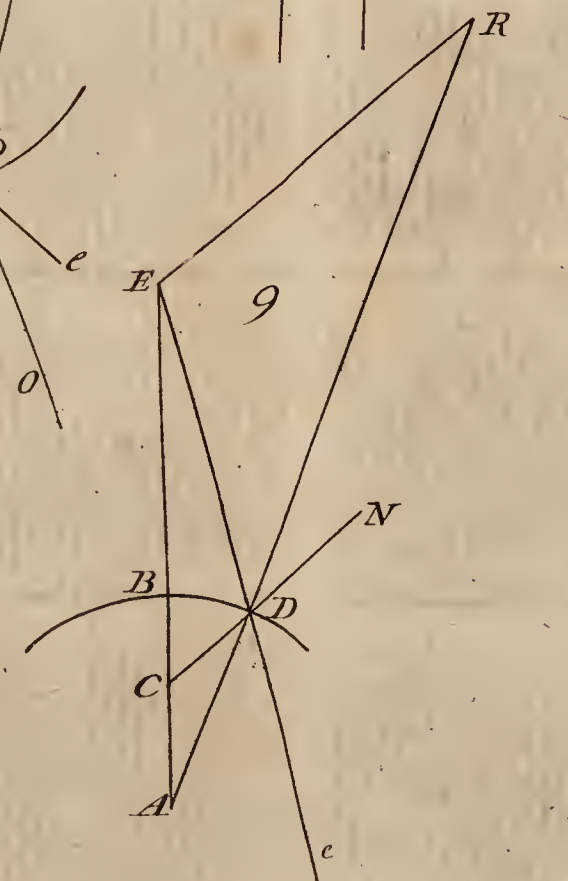
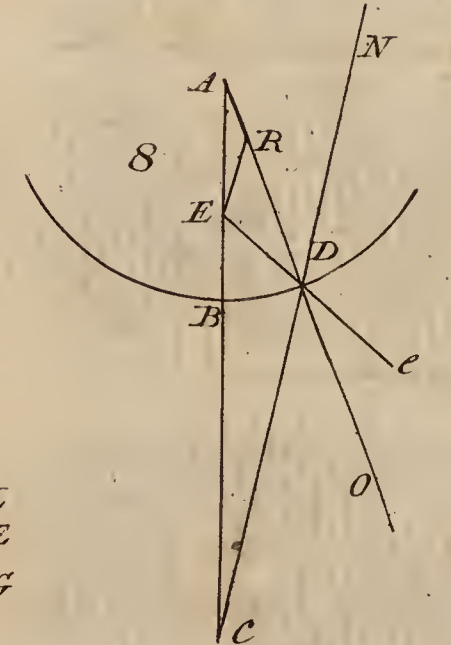
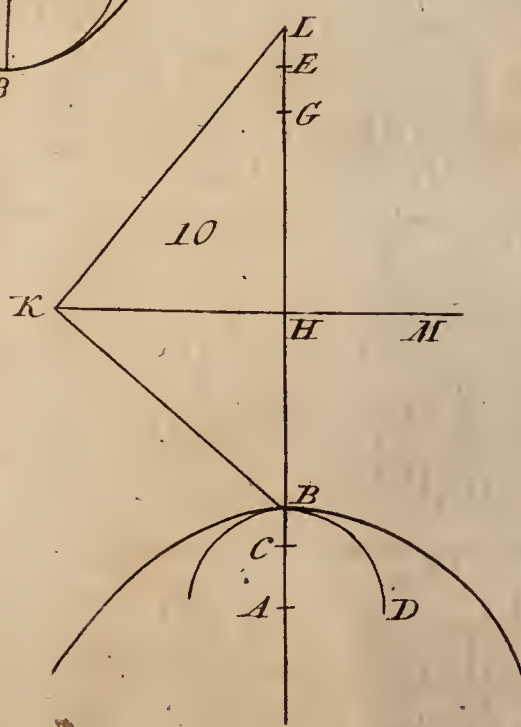
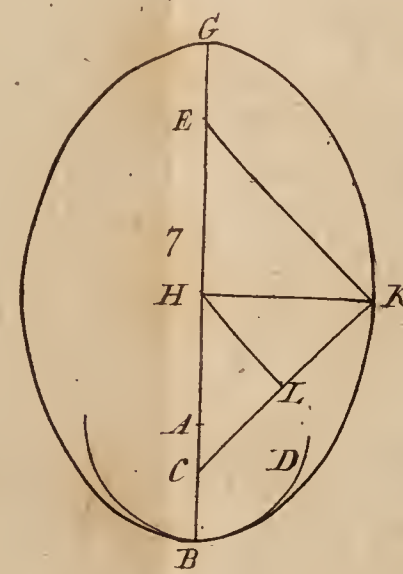
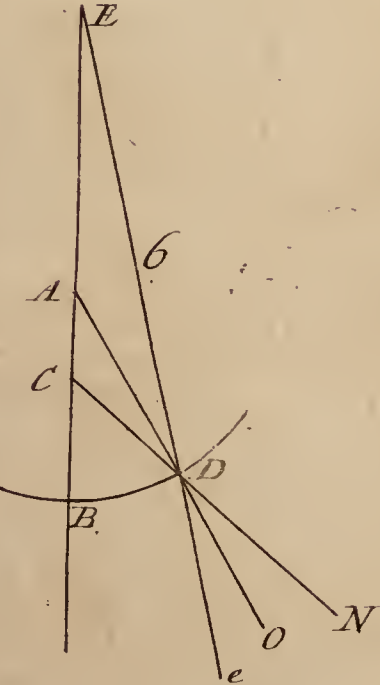
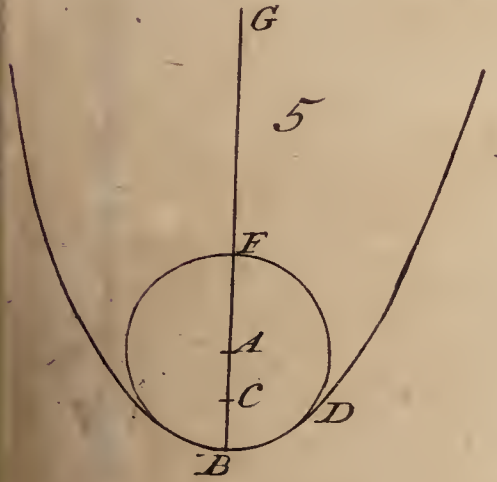
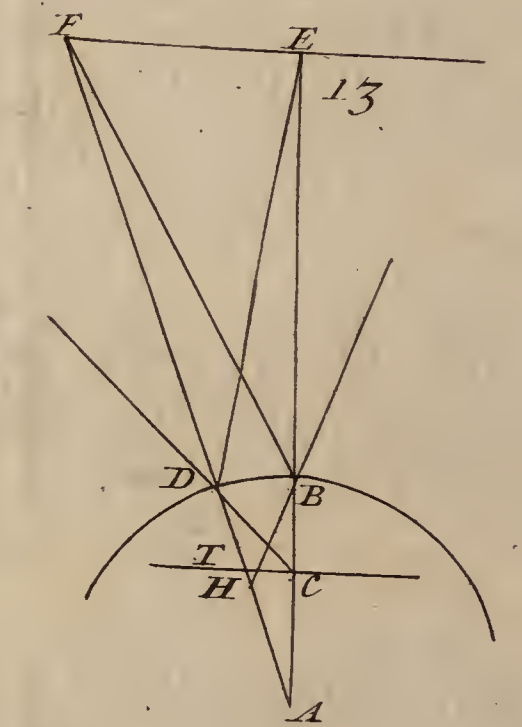
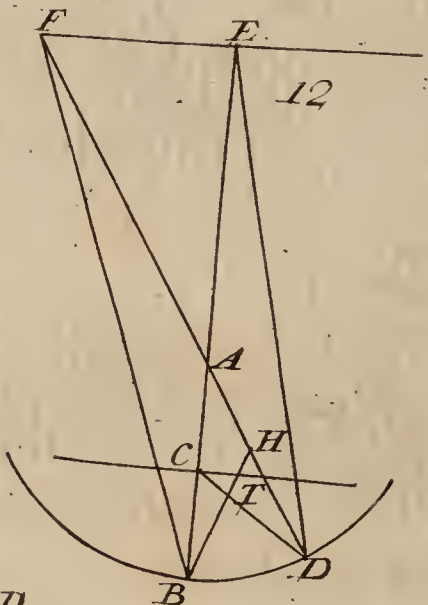
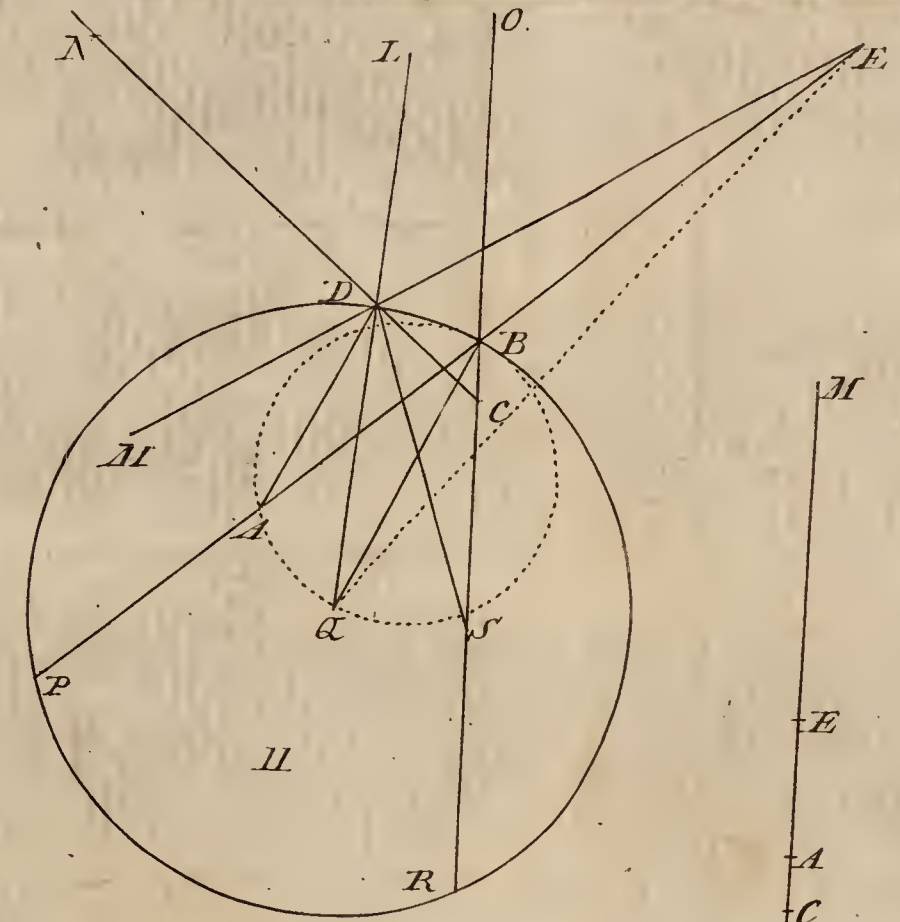
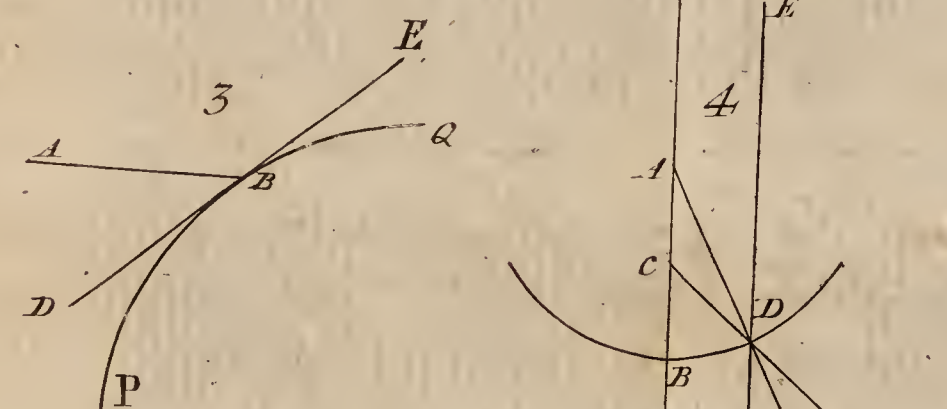
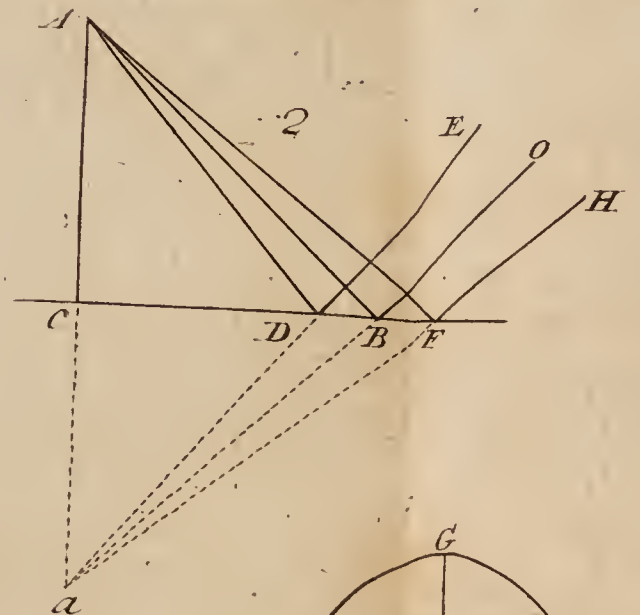
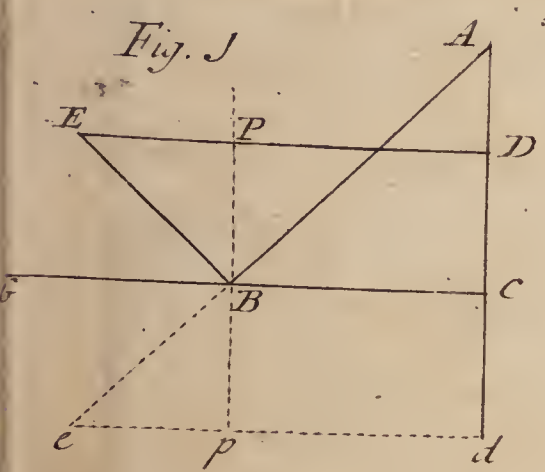
Errata & Corrigenda.

Pag. Line	For	Read
202 15	less	more.
206 1	after PR. OF. 6. refer to	Plate III.
207 8	Lens's	Lenses.

N. B. For want of Room, for the *sixth* Figure, (in PLATE IV.) the Rays O O and P P seem parallel to one another ; but they must be supposed to have cross'd at the Center of the *Speculum* upon the *Axis A a* produc'd, as is represented by the Rays *q O*, *q C*, instead of O O ; for the Rays O O, do really come from the Bottom of the *distant* Object *p o*, at *o*, and the Rays P P from the Top of the said Object at *p*.

These Errata & Corrigenda render this Work compleat.

J. T. D.



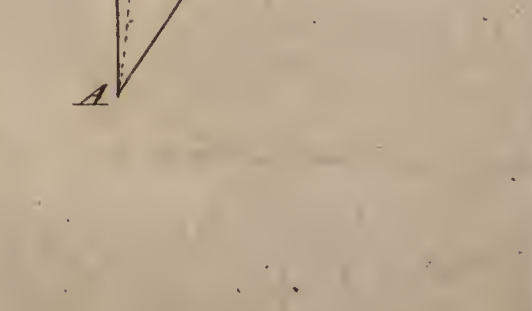
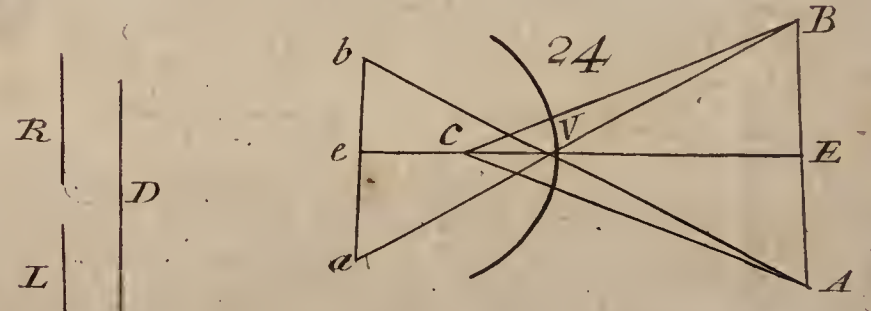
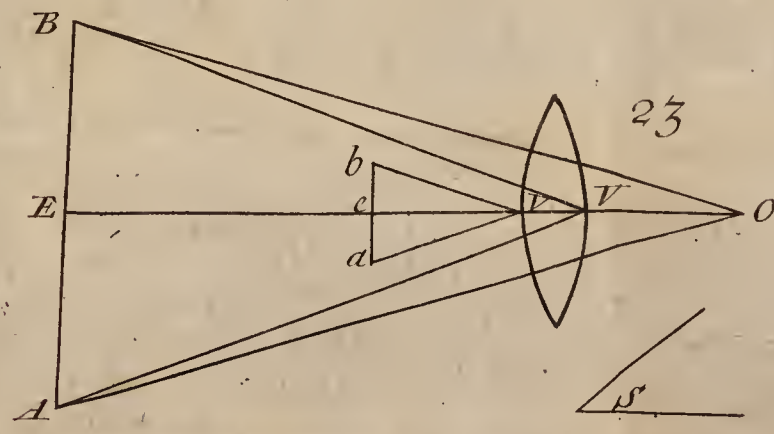
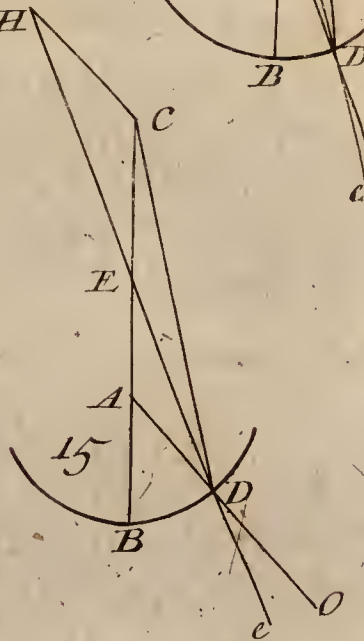
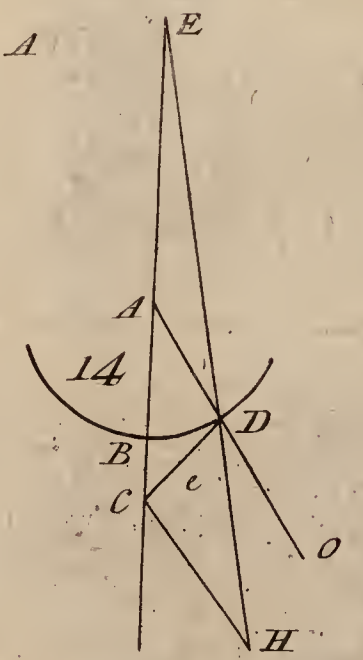
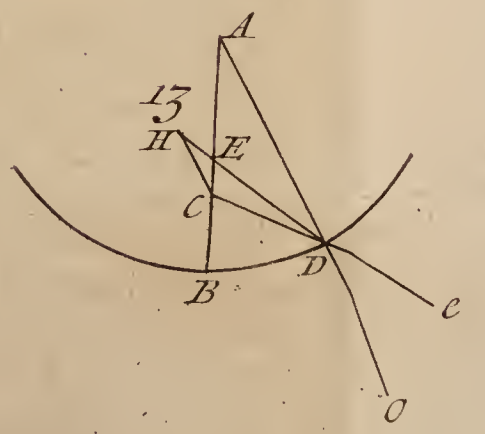
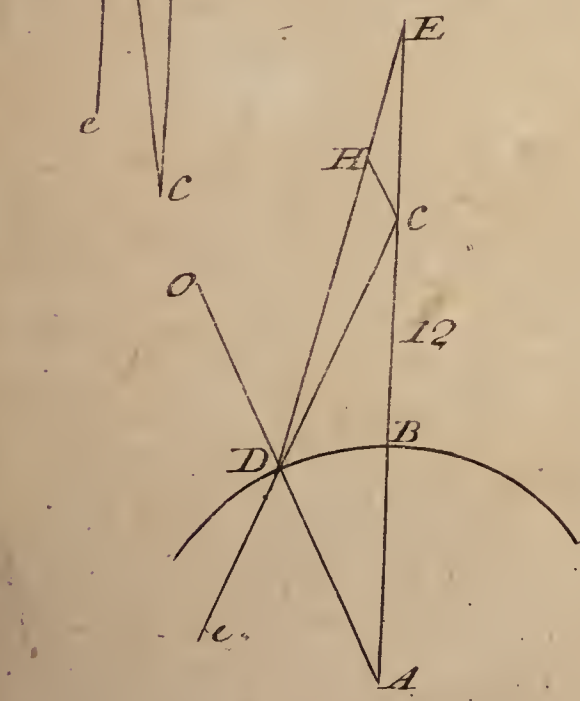
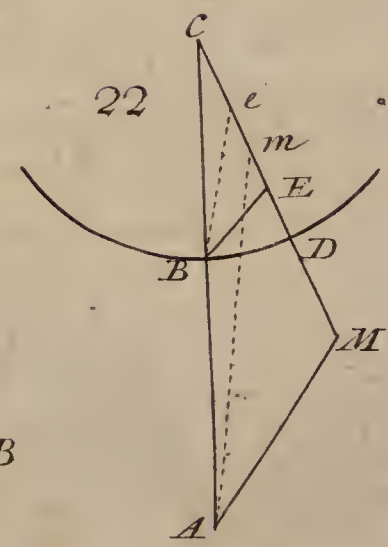
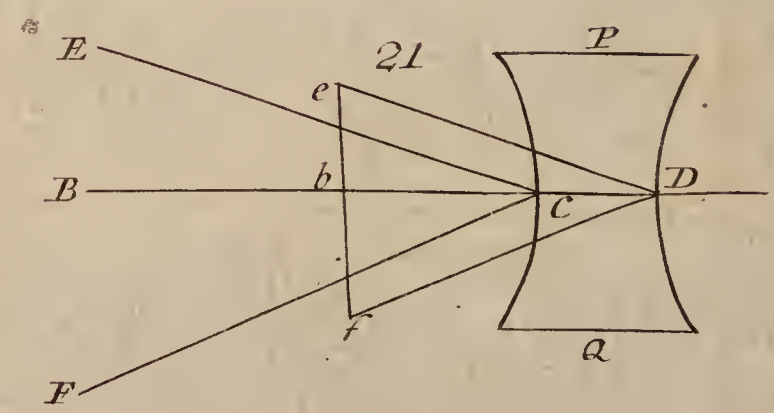
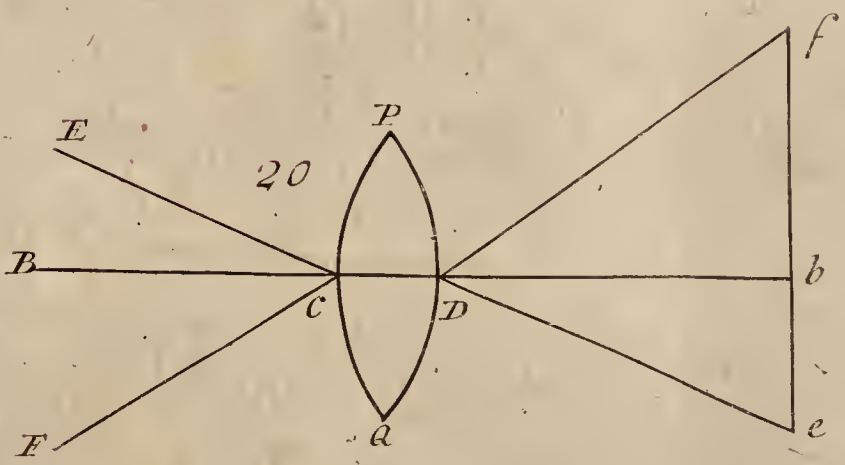
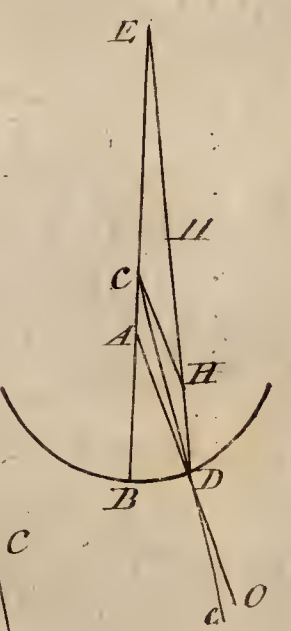
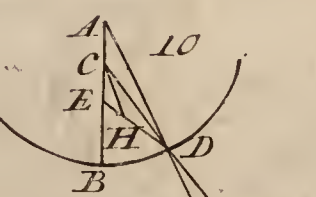
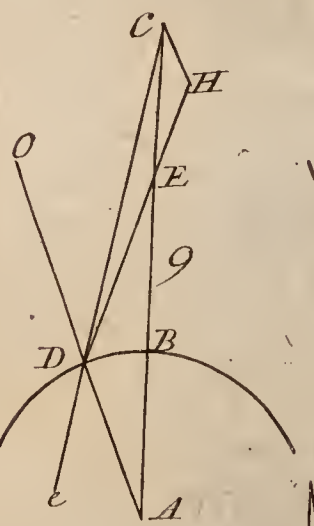
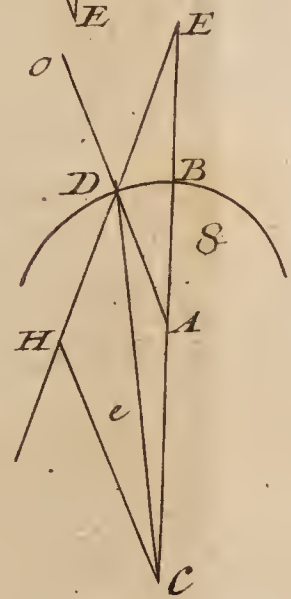
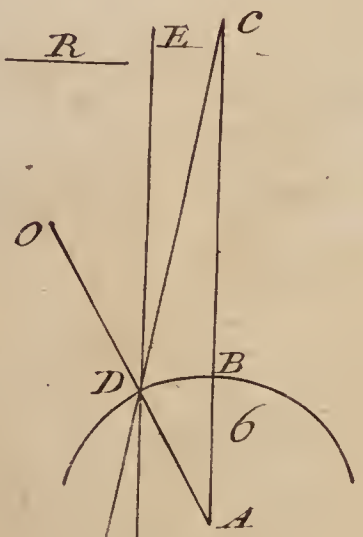
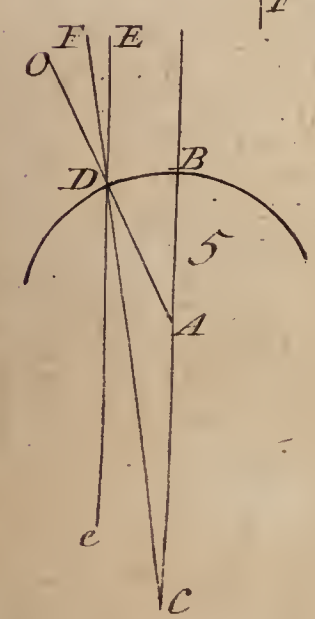
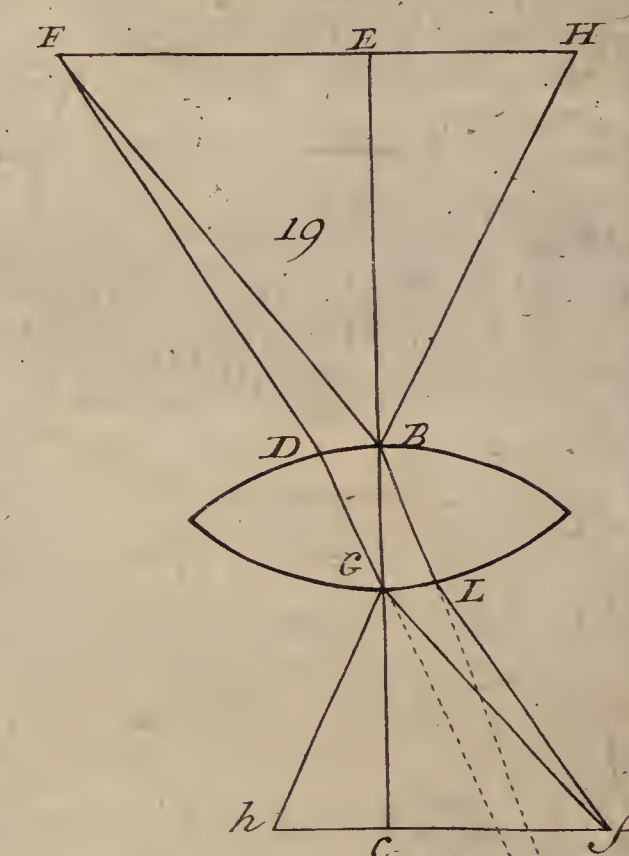
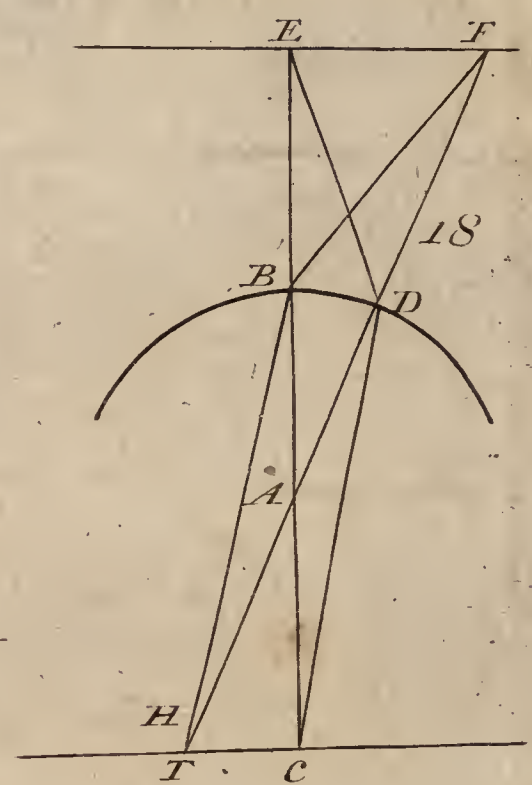
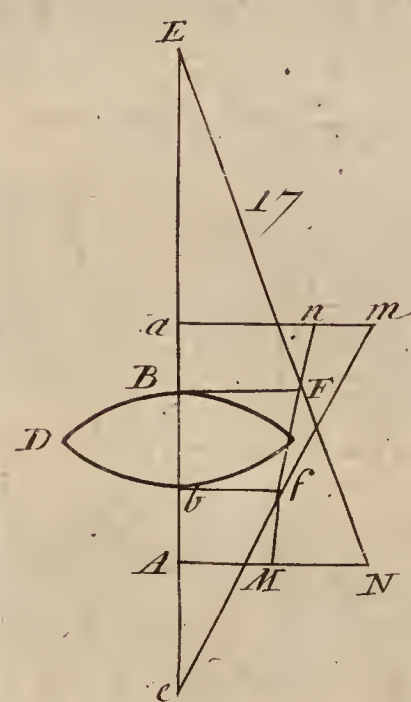
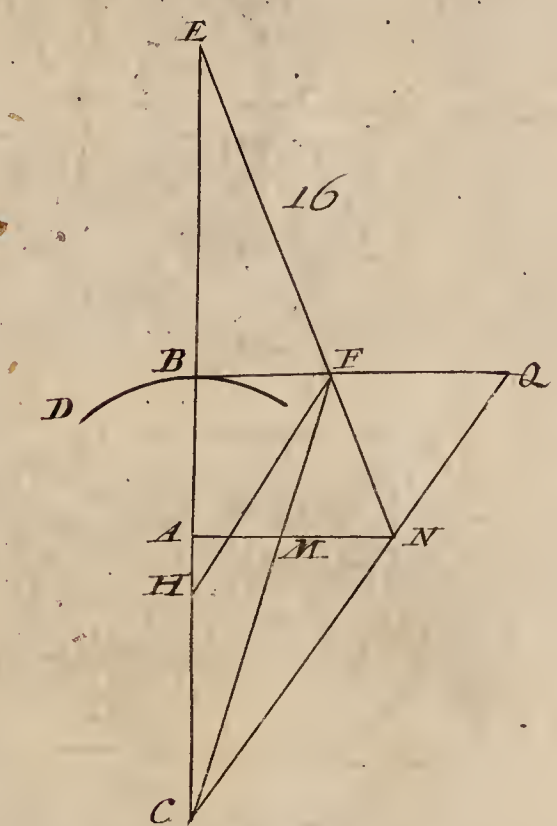
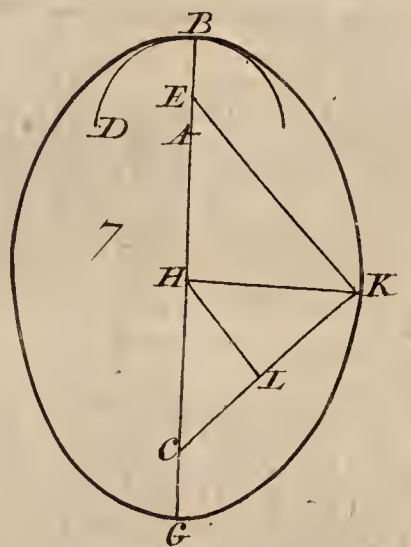
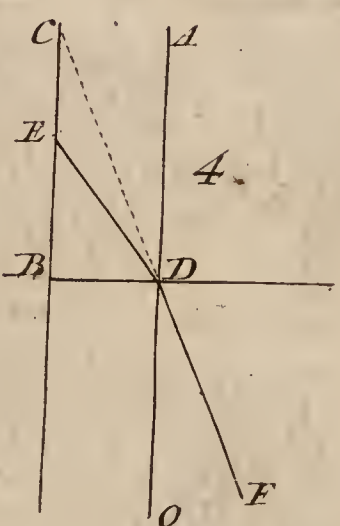
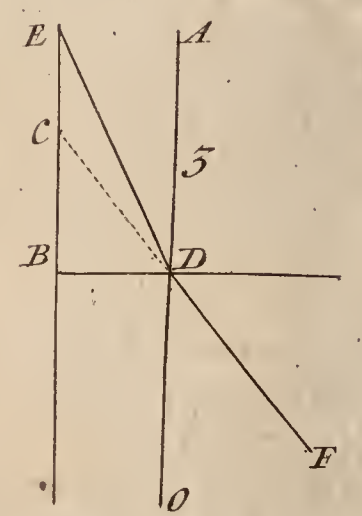
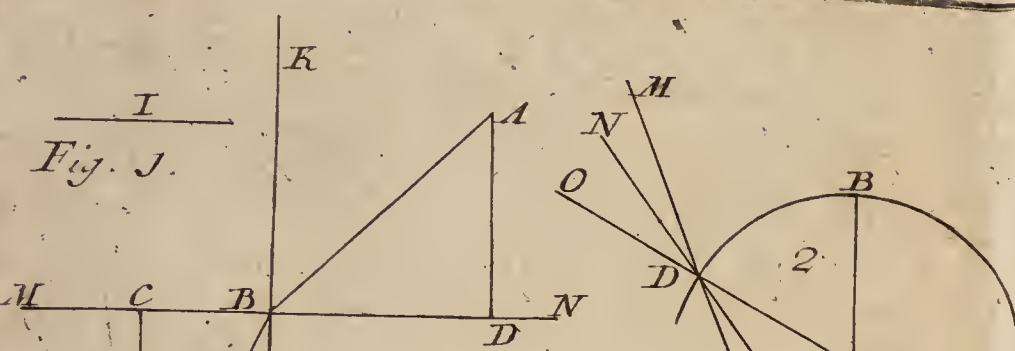




Plate III

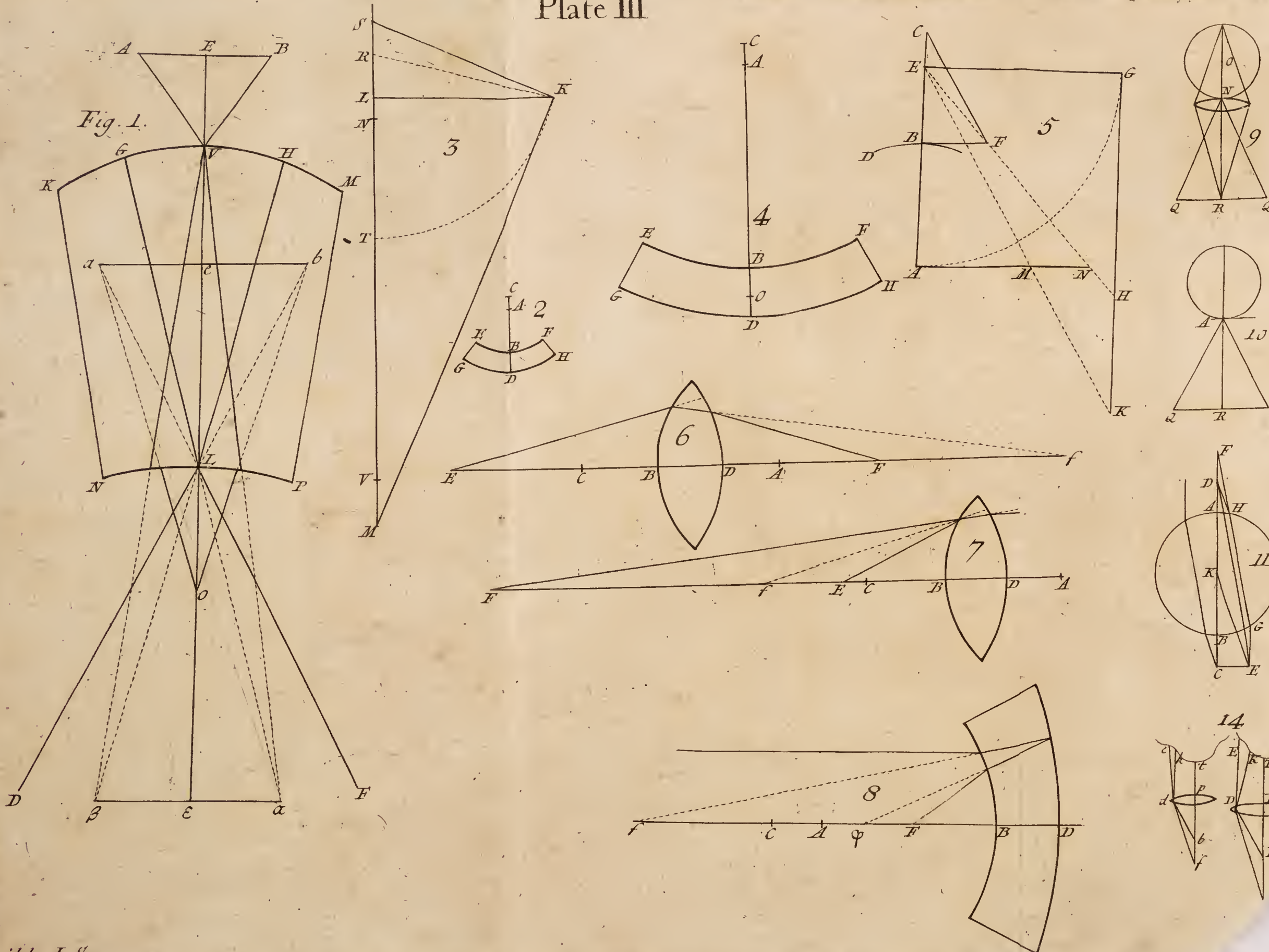


Fig. 3.



Fig. 4.

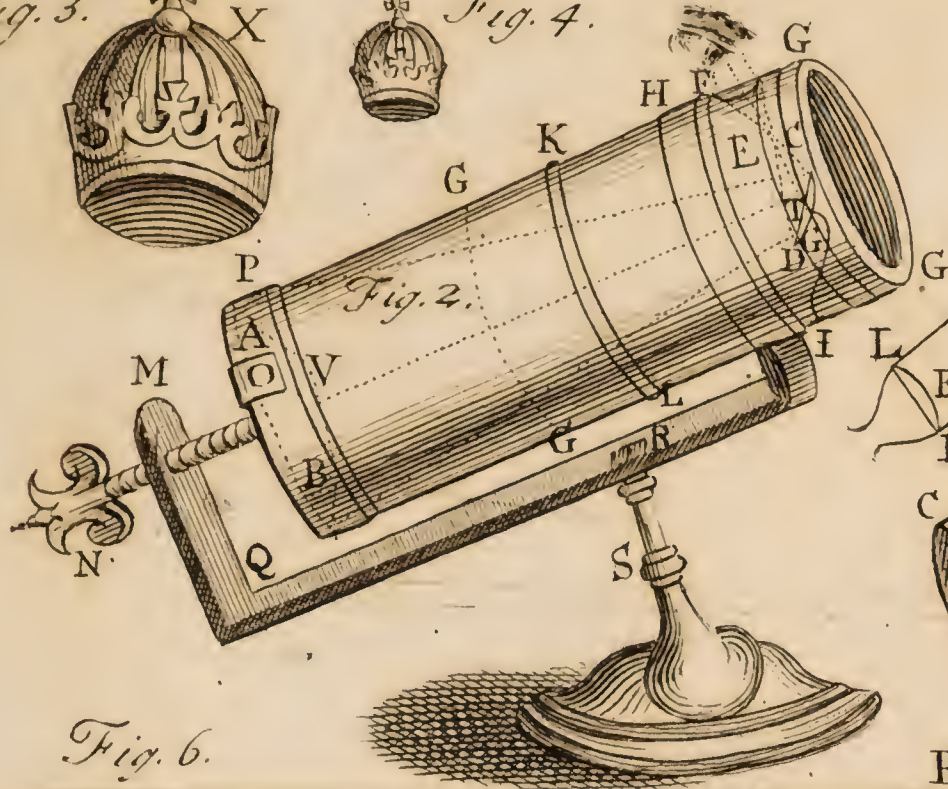


Fig. 1.

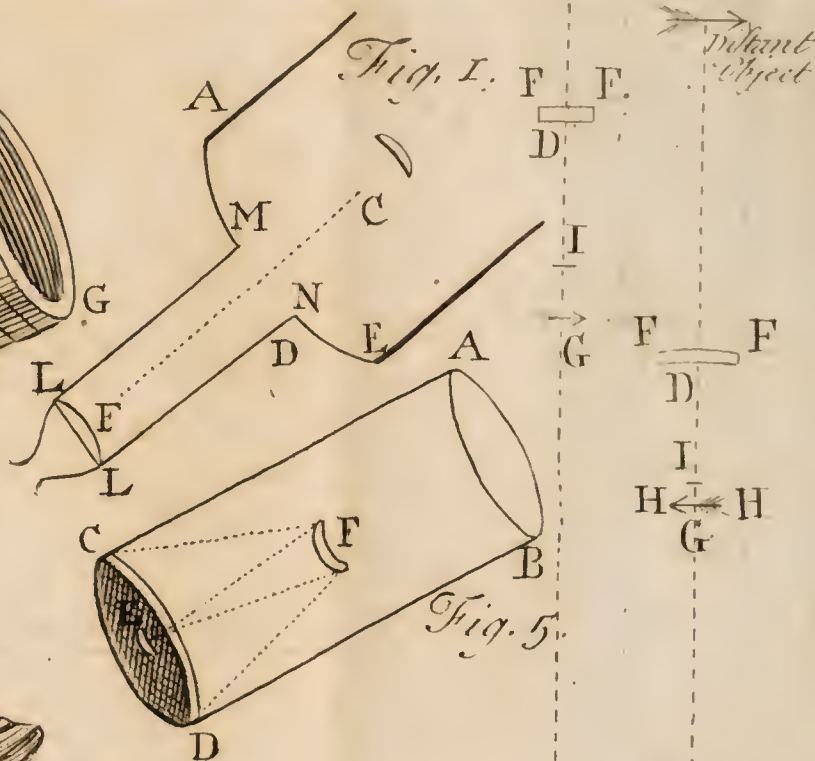


Fig. 6.

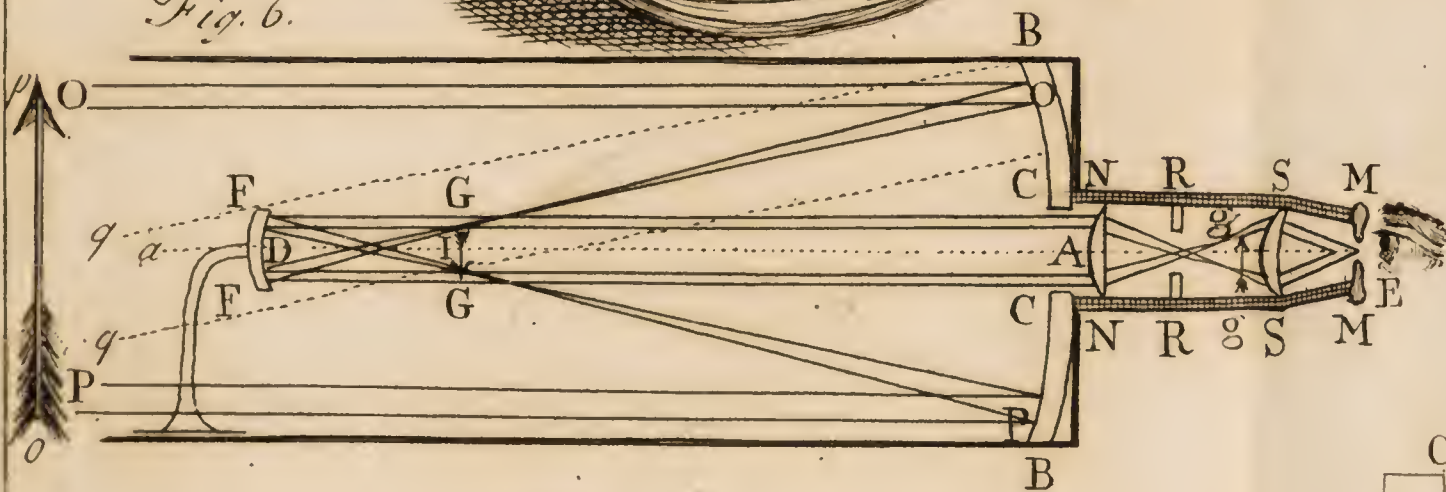


Fig. 7.

Fig. 8.

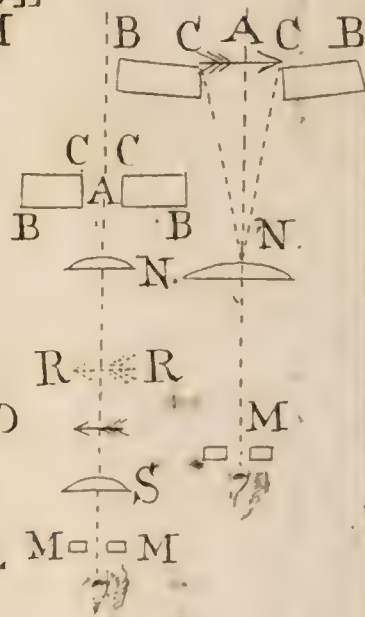


Fig. 9.

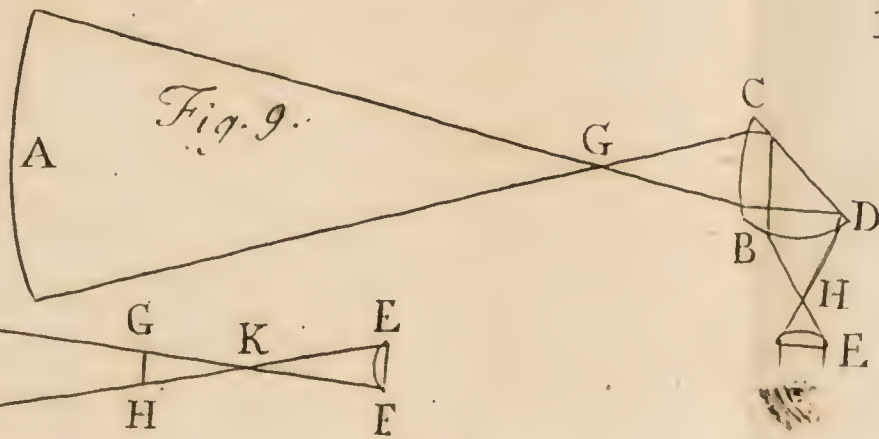


Fig. 10.

